# Moderne Methoden der Datenverarbeitung in der Physik

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- **Fehlerfortpflanzung I**
- **Fehlerfortpflanzung II, Zufallszahlen**
- **Maximum-Likelihood Methode, Gewichteter Mittelwert**
- **Methode der kleinsten Quadrate, Lineare Regression I**
- **Methode der kleinsten Quadrate, Lineare Regression II**
- **Fouriertransformation**

# Maximum-Likelihood-Methode ⇔ kleinste Quadrate

$$F(a) = -\ln L(a) = -\sum_{i=1}^{n} \ln f(x_i \mid a) = \text{Minimum} \Leftrightarrow \frac{\partial \mathrm{F}}{\partial \mathrm{a_k}} = 0 \text{ und } \frac{\partial^2 \mathrm{F}}{\partial \mathrm{a_k^2}} > 0$$

**Mittelwert einer Gauß-Verteilung**

**Die Gauß-Wahrscheinlichkeitsdichte ist**

$$f(x_i \mid a) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[ -\frac{(x_i - a)^2}{2\sigma_i^{\,2}} \right]$$

**mit dem Mittelwert a. Die negative Log-Likelihood-Funktion ist**

$$F(a) = konst.(\sigma_i) + \frac{1}{2} \sum_{i=1}^{n} \frac{(x_i - a)^2}{\sigma_i^2}$$

**Das Minimum von F(a) ist gleichbedeutend mit dem Minimum von:**

$$\chi^2 = \sum_{i=1}^{n} \frac{(x_i - a)^2}{\sigma_i^2}$$

**Entwickelt 1801 von C.F. Gauss für die Astronomie und Landvermessung**

# Methode der kleinste Quadrate

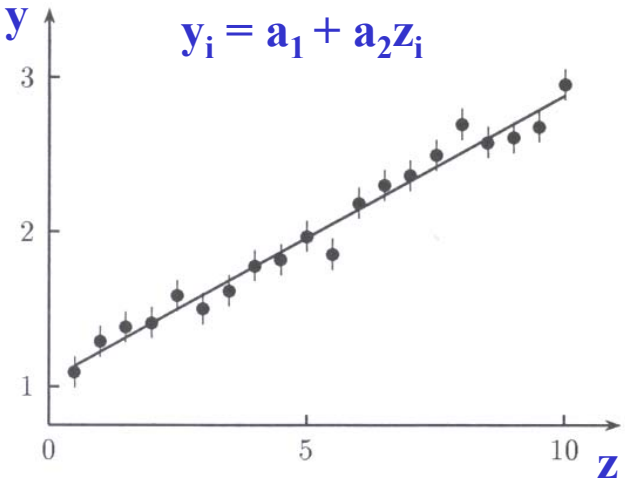
$$\chi^2 = \sum_{i=1}^{n} \frac{(x_i - a)^2}{\sigma_i^2}$$

**An einer Reihe von Stützstellen $z_i$ seien Werte für $y_i$ vorgeben. Theoretisch vorhergesagt seien $y_t(z_i|a)$.**

$$\chi^2 = \sum_{i=1}^{n} \frac{(y_i(z_i) - y_t(z_i \mid a))^2}{\sigma_i^2} = Minimum \Leftrightarrow \frac{d\chi^2}{da} = 0$$

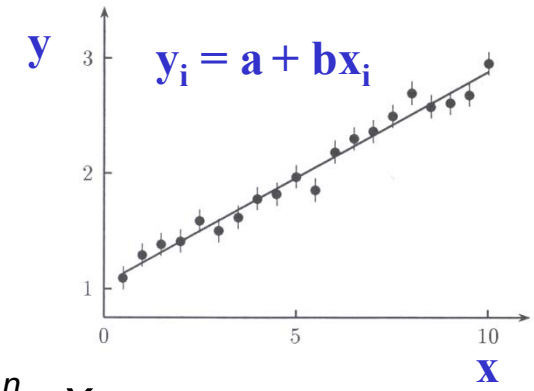
**Die Methode kann auf N Parameter $a_k$ erweitert werden:**

$$\frac{\partial \chi^2}{\partial a_k} = 0 \text{ für k=1,2,...,N}$$



# Anpassung einer Geraden

Geradengleichung: $y(x) = y(x \mid a,b) = a + b \cdot x$




$$\chi^2(a,b) = \sum_{i=1}^{n} \frac{(y_i - a - bx_i)^2}{\sigma_i^2}$$

Das Minimum von $\chi^2(a,b)$ wird bestimmt durch:

$$\frac{\partial \chi^2}{\partial \mathrm{a}} = 0 = \sum_{i=1}^{n} \frac{y_i - a - bx_i}{\sigma_i^2} \quad \Leftrightarrow \sum_{i=1}^{n} \frac{y_i}{\sigma_i^2} = a \sum_{i=1}^{n} \frac{1}{\sigma_i^2} + b \sum_{i=1}^{n} \frac{x_i}{\sigma_i^2}$$

$$\frac{\partial \chi^2}{\partial b} = 0 = \sum_{i=1}^{n} \frac{x_i (y_i - a - bx_i)}{\sigma_i^2} \quad \Leftrightarrow \sum_{i=1}^{n} \frac{x_i y_i}{\sigma_i^2} = a \sum_{i=1}^{n} \frac{x_i}{\sigma_i^2} + b \sum_{i=1}^{n} \frac{x_i x_i}{\sigma_i^2}$$

mit den Abkürzungen:

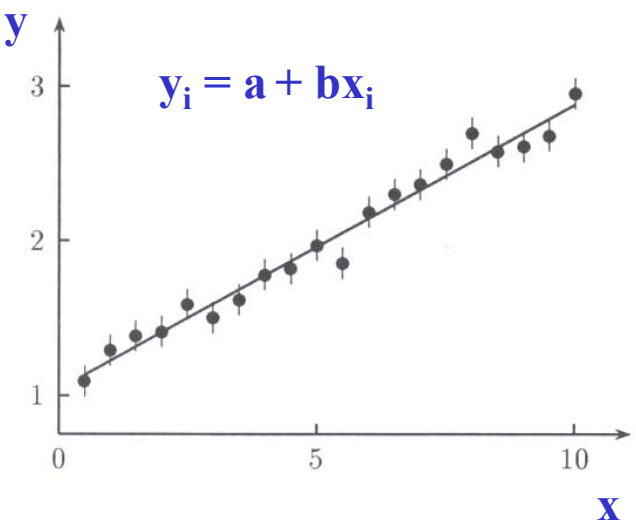
$$S_y \equiv \sum_{i=1}^{n} \frac{y_i}{\sigma_i^2} \qquad S_x \equiv \sum_{i=1}^{n} \frac{x_i}{\sigma_i^2} \qquad S_{xy} \equiv \sum_{i=1}^{n} \frac{x_i y_i}{\sigma_i^2} \qquad S_{xx} = \sum_{i=1}^{n} \frac{x_i x_i}{\sigma_i^2} \qquad S \equiv \sum_{i=1}^{n} \frac{1}{\sigma_i^2}$$

vereinfacht sich das Gleichungssystem zu:

$$S_y = a \cdot S + b \cdot S_x \quad \text{und} \quad S_{xy} = a \cdot S_x + b \cdot S_{xx}$$

# Anpassung einer Geraden

$S_y = a \cdot S + b \cdot S_x$ und $S_{xy} = a \cdot S_x + b \cdot S_{xx}$

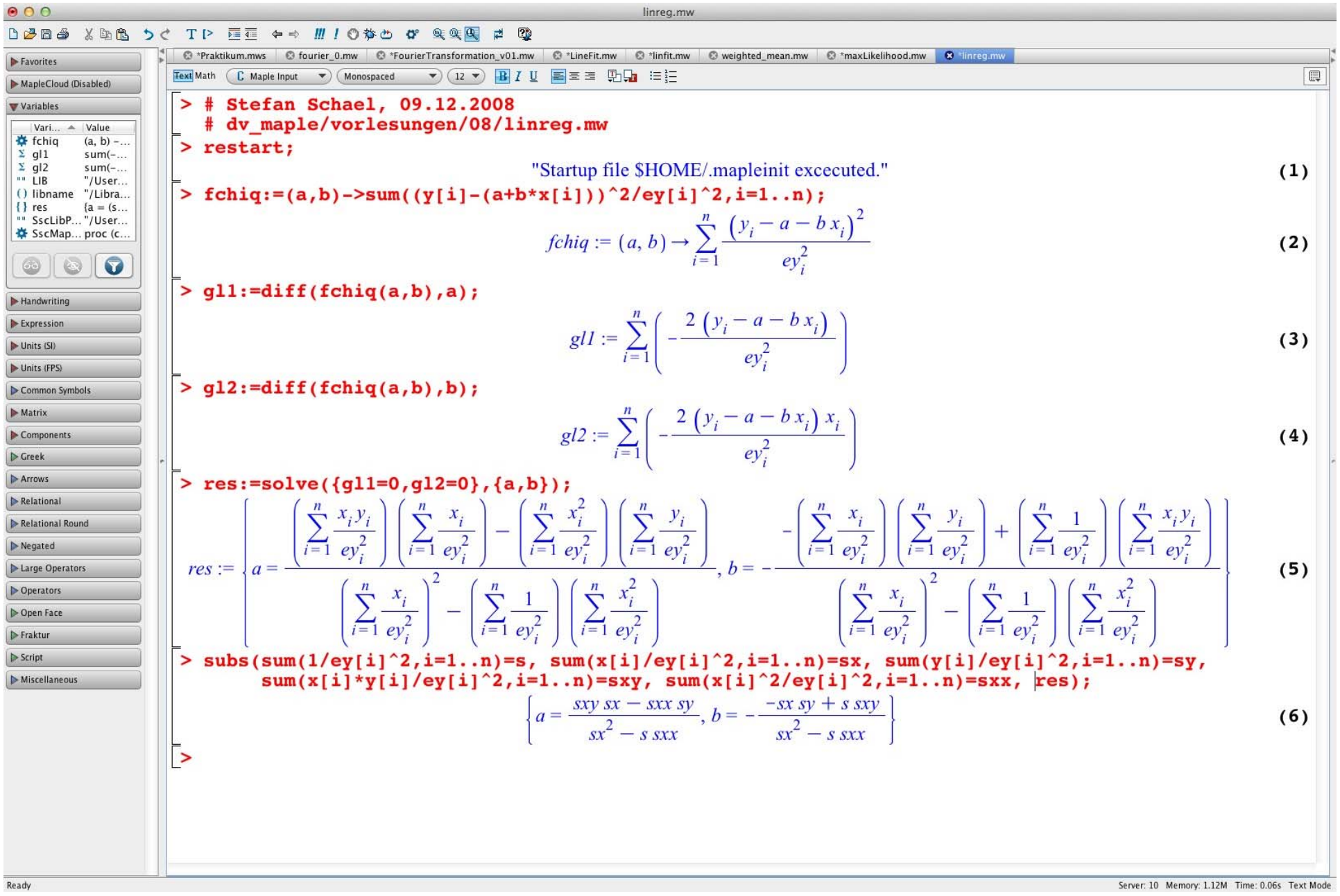



2. Gleichungen mit 2. Unbekannten: a,b

$$a = \frac{S_{xx} S_y - S_x S_{xy}}{S \cdot S_{xx} - S_x S_x} \quad \text{und} \quad b = \frac{S \cdot S_{xy} - S_x S_y}{S \cdot S_{xx} - S_x S_x}$$

$$S_y \equiv \sum_{i=1}^{n} \frac{y_i}{\sigma_i^2} \qquad S_x \equiv \sum_{i=1}^{n} \frac{x_i}{\sigma_i^2} \qquad S_{xy} \equiv \sum_{i=1}^{n} \frac{x_i y_i}{\sigma_i^2}$$

$$S_{xx} = \sum_{i=1}^{n} \frac{x_i x_i}{\sigma_i^2} \qquad S \equiv \sum_{i=1}^{n} \frac{1}{\sigma_i^2} \qquad \Delta = S \cdot S_{xx} - S_x S_x$$


linreg.mw
*Praktikum.mws
fourier_0.mw
*FourierTransformation_v01.mw
*LineFit.mw
*linfit.mw
weighted_mean.mw
*maxLikelihood.mw
*linreg.mw
Favorites
MapleCloud (Disabled)
Variables
Handwriting
Expression
Units (SI)
Units (FPS)
Common Symbols
Matrix
Components
Greek
Arrows
Relational
Relational Round
Negated
Large Operators
Operators
Open Face
Fraktur
Script
Miscellaneous
> # Stefan Schael, 09.12.2008
# dv_maple/vorlesungen/08/linreg.mw
> restart;
"Startup file $HOME/.mapleinit excecuted."
(1)
> fchiq:=(a,b)->sum((y[i]-(a+b*x[i]))^2/ey[i]^2,i=1..n);
(2)
> gl1:=diff(fchiq(a,b),a);
(3)
> gl2:=diff(fchiq(a,b),b);
(4)
> res:=solve({gl1=0,gl2=0},{a,b});
(5)
> subs(sum(1/ey[i]^2,i=1..n)=s, sum(x[i]/ey[i]^2,i=1..n)=sx, sum(y[i]/ey[i]^2,i=1..n)=sy,
sum(x[i]*y[i]/ey[i]^2,i=1..n)=sxy, sum(x[i]^2/ey[i]^2,i=1..n)=sxx, res);
(6)
Ready
Server: 10 Memory: 1.12M Time: 0.06s Text Mode

# Anpassung einer Geraden

**Was sind die Fehler von a und b ?**

$$a = a(y_i) \Rightarrow \sigma_a^2 = \sum_{i=1}^{n} \sigma_i^2 \left( \frac{\partial a}{\partial y_i} \right)^2 \quad \text{und entsprechend} \quad \sigma_b^2 = \sum_{i=1}^{n} \sigma_i^2 \left( \frac{\partial b}{\partial y_i} \right)^2$$

$$\frac{\partial a}{\partial y_i} = \frac{1}{\Delta}\left( S_{xx} \frac{\partial S_y}{\partial y_i} - S_x \frac{\partial S_{xy}}{\partial y_i} \right) = \frac{1}{\Delta}\left( S_{xx} \frac{1}{\sigma_i^2} - S_x \frac{x_i}{\sigma_i^2} \right) = \frac{1}{\Delta \sigma_i^2}\left( S_{xx} - S_x \cdot x_i \right)$$

$$\frac{\partial b}{\partial y_i} = \frac{1}{\Delta}\left( S \frac{\partial S_{xy}}{\partial y_i} - S_x \frac{\partial S_y}{\partial y_i} \right) = \frac{1}{\Delta}\left( S \frac{x_i}{\sigma_i^2} - S_x \frac{1}{\sigma_i^2} \right) = \frac{1}{\Delta \sigma_i^2}\left( S \cdot x_i - S_x \right)$$

$$\Rightarrow \quad \sigma_a^2 = \frac{S_{xx}}{\Delta} = \frac{S_{xx}}{S \cdot S_{xx} - S_x S_x}$$

$$\sigma_b^2 = \frac{S}{\Delta} = \frac{S}{S \cdot S_{xx} - S_x S_x}$$

$$S_y \equiv \sum_{i=1}^{n} \frac{y_i}{\sigma_i^2} \qquad S_x \equiv \sum_{i=1}^{n} \frac{x_i}{\sigma_i^2} \qquad S_{xy} \equiv \sum_{i=1}^{n} \frac{x_i y_i}{\sigma_i^2}$$

$$S_{xx} = \sum_{i=1}^{n} \frac{x_i x_i}{\sigma_i^2} \qquad S \equiv \sum_{i=1}^{n} \frac{1}{\sigma_i^2} \qquad \Delta = S \cdot S_{xx} - S_x S_x$$

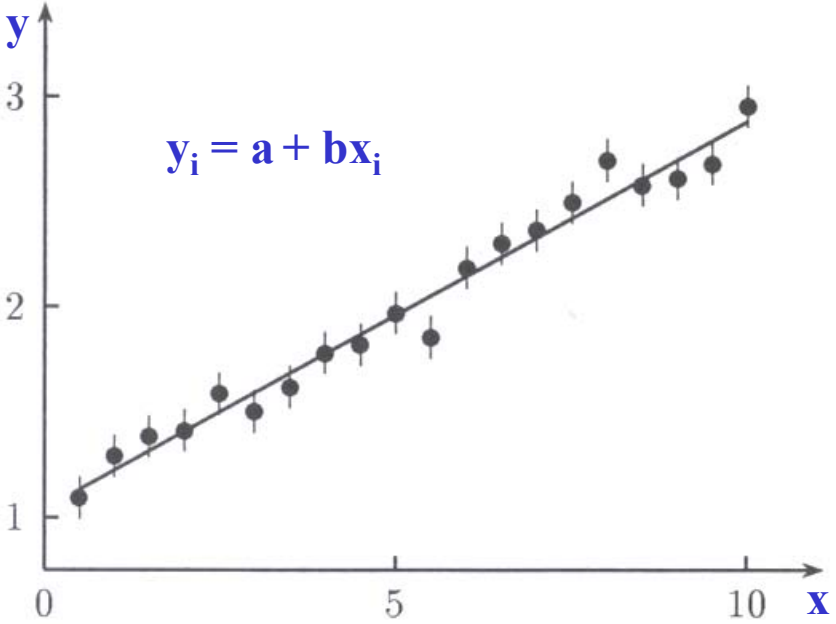
# Anpassung einer Geraden

**Was ist die Kovarianz zwischen a und b ?**

$$\sigma_{ab} = Cov(a,b) = \sum_{i=1}^{n} \sigma_i^2 \frac{\partial a}{\partial y_i}\frac{\partial b}{\partial y_i} = -\frac{S_x}{\Delta} = -\frac{S_x}{S\cdot S_{xx} - S_x S_x}$$

und der Korrelationskoeffizient ist:

$$\rho(a,b) = \frac{\sigma_{ab}}{\sigma_a \sigma_b} = \frac{-S_x}{\sqrt{S\cdot S_{xx}}}$$




$$S_y \equiv \sum_{i=1}^{n} \frac{y_i}{\sigma_i^2} \qquad S_x \equiv \sum_{i=1}^{n} \frac{x_i}{\sigma_i^2} \qquad S_{xy} \equiv \sum_{i=1}^{n} \frac{x_i y_i}{\sigma_i^2}$$

$$S_{xx} = \sum_{i=1}^{n} \frac{x_i x_i}{\sigma_i^2} \qquad S \equiv \sum_{i=1}^{n} \frac{1}{\sigma_i^2} \qquad \Delta = S\cdot S_{xx} - S_x S_x$$

$$a = \frac{S_{xx}S_y - S_x S_{xy}}{\Delta} \text{ und } b = \frac{S\cdot S_{xy} - S_x S_y}{\Delta}$$

$$\sigma_a^2 = \frac{S_{xx}}{\Delta} \qquad \sigma_b^2 = \frac{S}{\Delta} \qquad \rho(a,b) = \frac{-S_x}{\sqrt{S\cdot S_{xx}}}$$

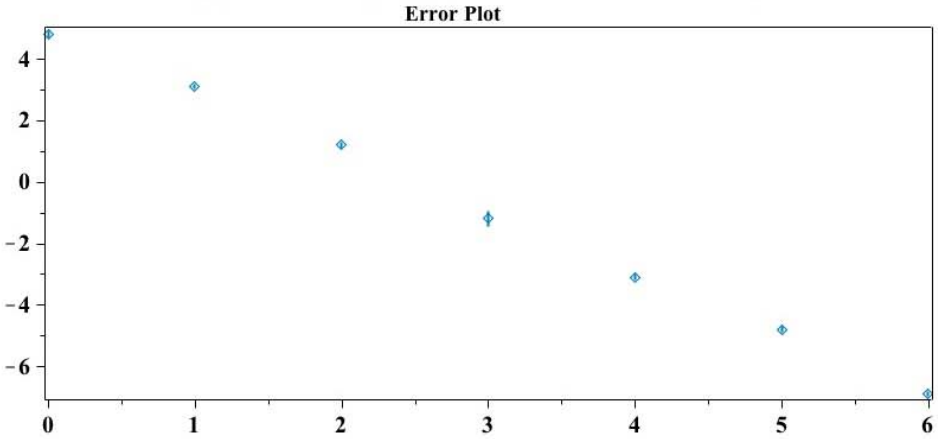
LineFit.mw

*Praktikum.mws | fourier_0.mw | *FourierTransformation_v01.mw | *LineFit.mw

```
> # Stefan Schael, 20.05.2012
  # dv_maple/vorlesung/08/linfit.mw
> restart:
```

"Startup file $HOME/.mapleinit excecuted." (1)

```
> with(plots): with(Statistics): with(plottools):
> xvec := [0,1,2,3,4,5,6]:
> yvec := [4.8, 3.1, 1.2, -1.2, -3.1, -4.8, -6.9]:
> sig  := [0.13,0.08,0.1,0.25,0.09,0.1,0.06]:
> ErrorPlot(yvec, xcoords=xvec,yerrors=sig, title = "Error Plot");
```




```
> sx  := sum(xvec[k]/sig[k]^2,k=1..nops(xvec)):
  sy  := sum(yvec[k]/sig[k]^2,k=1..nops(xvec)):
  s   := sum(1/sig[k]^2,k=1..nops(sig)):
  sxx := sum(xvec[k]^2/sig[k]^2,k=1..nops(xvec)):
  sxy := sum(xvec[k]*yvec[k]/sig[k]^2,k=1..nops(xvec)):
> a   := (sxx*sy-sx*sxy)/(s*sxx-sx*sx);
  sa  := evalf(sqrt(sxx/(s*sxx-sx*sx)));
```

$a := 5.013320255$

$sa := 0.06840285787$ (2)

```
> m   := (s*sxy-sx*sy)/(s*sxx-sx*sx);
  sm  := evalf(sqrt(s/(s*sxx-sx*sx)));
```

$m := -1.985339202$

$sm := 0.01602266507$ (3)

Ready

Server: 6 Memory: 1.12M Time: 0.07s Math Mode

LineFit.mw

*Praktikum.mws | fourier_0.mw | *FourierTransformation_v01.mw | *LineFit.mw

Text Math | Maple Plot | Times New Roman | 12

$a := 5.013320255$

$sa := 0.06840285787$ (2)

```
> m   := (s*sxy-sx*sy)/(s*sxx-sx*sx);
  sm  := evalf(sqrt(s/(s*sxx-sx*sx)));
```

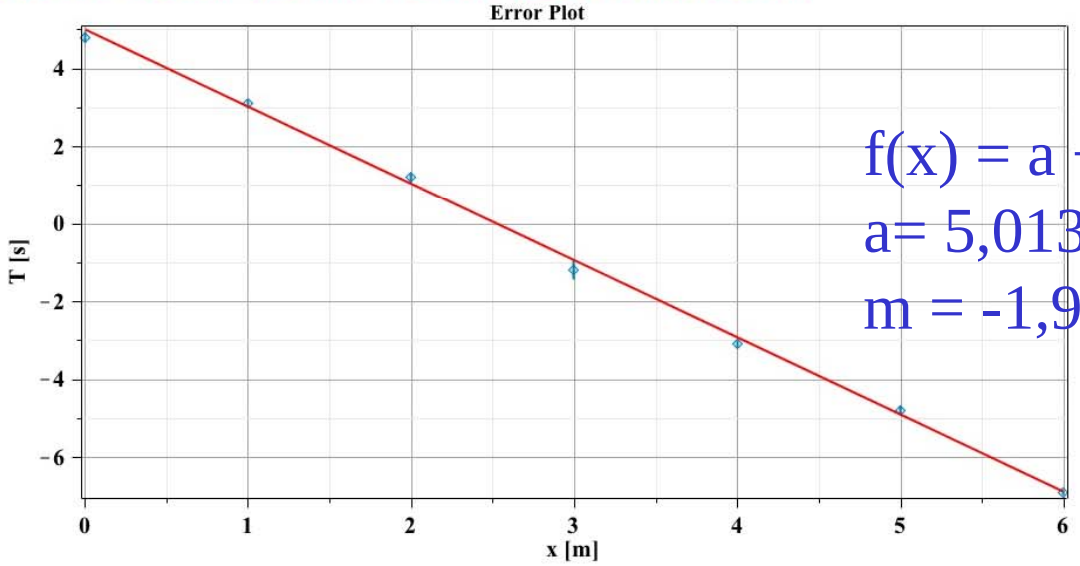
$m := -1.985339202$

$sm := 0.01602266507$ (3)

```
> f     := t->a+m*t;
```

$f := t \to a + m\,t$ (4)

```
> Plot1 := ErrorPlot(yvec, xcoords=xvec,yerrors=sig, title = "Error Plot"):
> Plot2 := plot(f(t),t=0..6):
> display(Plot1,Plot2,labels=["x [m]","T [s]"],gridlines=true);
```




f(x) = a + m·x

a= 5,013 ± 0,068

m = -1,985 ± 0,016

## Korrelation & Kovarianz-Matrix

```
> rho       := -sx/sqrt(s*sxx);
```

$\rho := -0.8621622122$

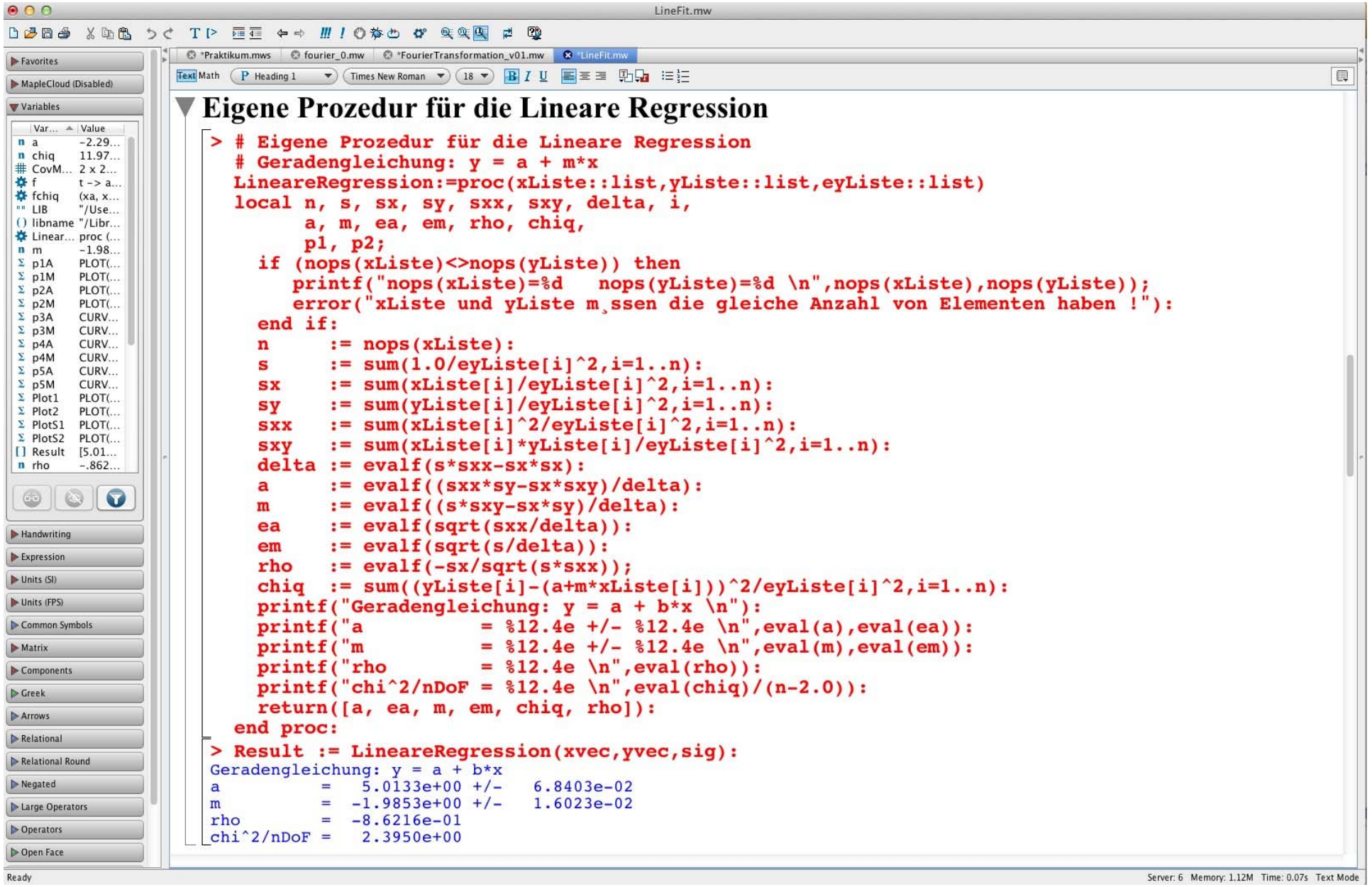
```
> CovMatrix := Matrix([[sa^2, rho*sa*sm],[rho*sa*sm,sm^2]]);
```

$$CovMatrix := \begin{bmatrix} 0.004678950965 & -0.0009449264061 \\ -0.0009449264061 & 0.0002567257959 \end{bmatrix}$$

$$\rho(a,m) = \frac{\sigma_{am}}{\sigma_a \sigma_m} = \frac{-S_x}{\sqrt{S \cdot S_{xx}}} \quad (1.1)$$ (1.2)

Ready

Server: 6 Memory: 1.0GM Time: 0.07s Text Mode


LineFit.mw
*Praktikum.mws
fourier_0.mw
*FourierTransformation_v01.mw
*LineFit.mw
Favorites
MapleCloud (Disabled)
Variables
Handwriting
Expression
Units (SI)
Units (FPS)
Common Symbols
Matrix
Components
Greek
Arrows
Relational
Relational Round
Negated
Large Operators
Operators
Open Face
Eigene Prozedur für die Lineare Regression
> # Eigene Prozedur für die Lineare Regression
# Geradengleichung: y = a + m*x
LineareRegression:=proc(xListe::list,yListe::list,eyListe::list)
local n, s, sx, sy, sxx, sxy, delta, i,
a, m, ea, em, rho, chiq,
p1, p2;
if (nops(xListe)<>nops(yListe)) then
printf("nops(xListe)=%d nops(yListe)=%d \n",nops(xListe),nops(yListe));
error("xListe und yListe m¸ssen die gleiche Anzahl von Elementen haben !"):
end if:
n := nops(xListe):
s := sum(1.0/eyListe[i]^2,i=1..n):
sx := sum(xListe[i]/eyListe[i]^2,i=1..n):
sy := sum(yListe[i]/eyListe[i]^2,i=1..n):
sxx := sum(xListe[i]^2/eyListe[i]^2,i=1..n):
sxy := sum(xListe[i]*yListe[i]/eyListe[i]^2,i=1..n):
delta := evalf(s*sxx-sx*sx):
a := evalf((sxx*sy-sx*sxy)/delta):
m := evalf((s*sxy-sx*sy)/delta):
ea := evalf(sqrt(sxx/delta)):
em := evalf(sqrt(s/delta)):
rho := evalf(-sx/sqrt(s*sxx)):
chiq := sum((yListe[i]-(a+m*xListe[i]))^2/eyListe[i]^2,i=1..n):
printf("Geradengleichung: y = a + b*x \n"):
printf("a = %12.4e +/- %12.4e \n",eval(a),eval(ea)):
printf("m = %12.4e +/- %12.4e \n",eval(m),eval(em)):
printf("rho = %12.4e \n",eval(rho)):
printf("chi^2/nDoF = %12.4e \n",eval(chiq)/(n-2.0)):
return([a, ea, m, em, chiq, rho]):
end proc:
> Result := LineareRegression(xvec,yvec,sig):
Geradengleichung: y = a + b*x
a = 5.0133e+00 +/- 6.8403e-02
m = -1.9853e+00 +/- 1.6023e-02
rho = -8.6216e-01
chi^2/nDoF = 2.3950e+00
Ready
Server: 6 Memory: 1.12M Time: 0.07s Text Mode

Nach der linearen Koordinatentransformation $x_i' = x_i - \frac{S_x}{S}$ gilt: $\rho = 0$

LineFit.mw

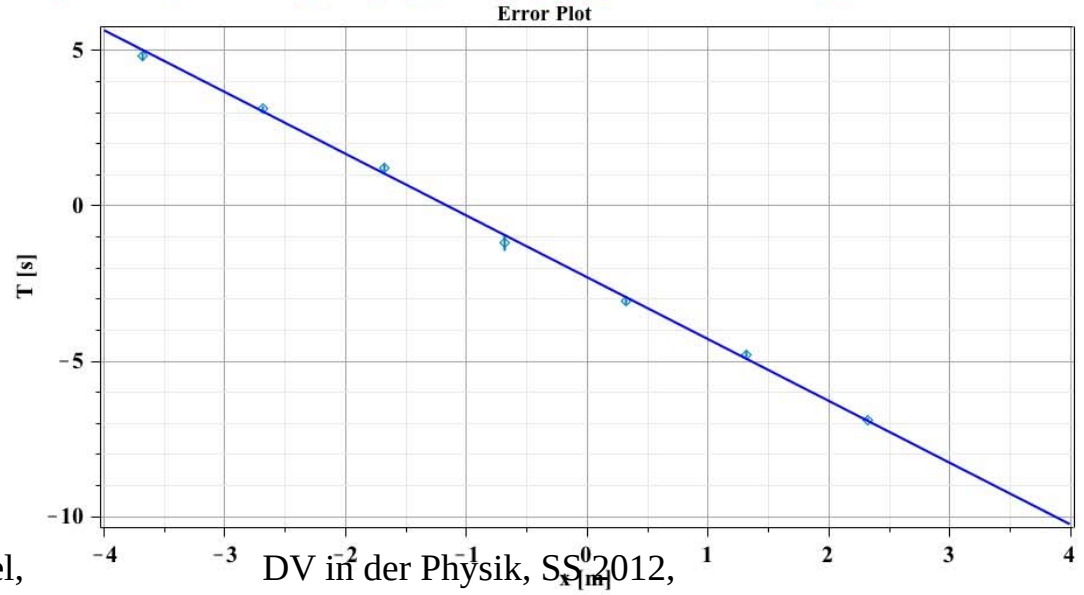
**Korrelation zwischen a und m durch Transformation minimieren**

```
> xvecS  := map(x->x-sx/s,xvec):
> Result := LineareRegression(xvecS,yvec,sig):
Geradengleichung: y = a + b*x
a           =  -2.2941e+00 +/-   3.4655e-02
m           =  -1.9853e+00 +/-   1.6023e-02
rho         =   0.0000e-01
chi^2/nDoF =   2.3950e+00
> a      := Result[1]:
  SigmaA := Result[2]:
  m      := Result[3]:
  SigmaM := Result[4]:
> PlotS1 := ErrorPlot(yvec, xcoords=xvecS,yerrors=sig, title = "Error Plot"):
> PlotS2 := plot(a+m*t,t=-4..4,color=blue):
> display(PlotS1,PlotS2,labels=["x [m]","T [s]"],gridlines=true);
```

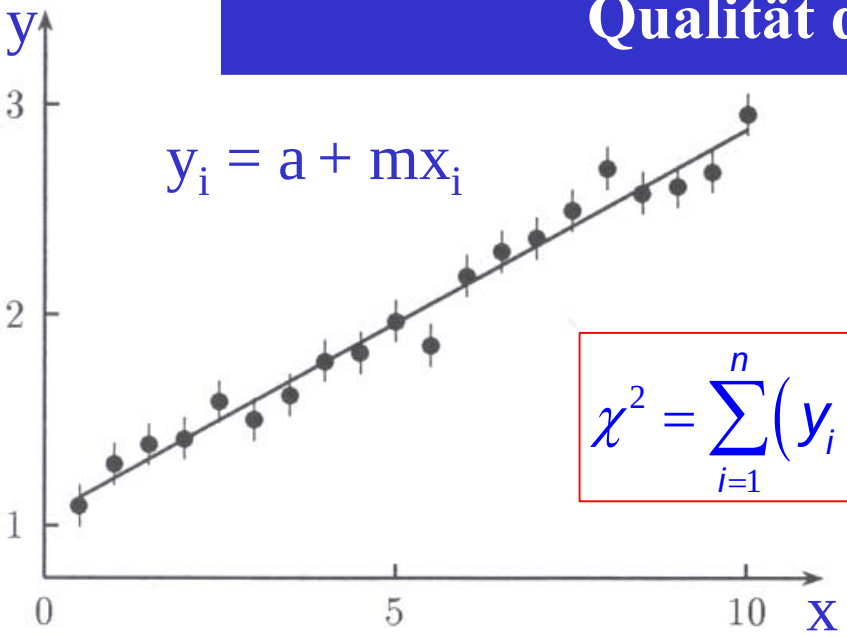
$f(x) = a + m \cdot x$

$a = 2{,}294 \pm 0{,}035$

$m = -1{,}985 \pm 0{,}016$



# Qualität der Anpassung




$$\chi^2 = \sum_{i=1}^{n} \left( y_i - f(x_i) \right)^2 / \sigma_i^2 = \sum_{i=1}^{n} \left( y_i - \left( a + m \cdot x_i \right) \right)^2 / \sigma_i^2$$

**Wie beurteile ich, ob die Gerade meine Messwerte gut beschreibt ? <= $\chi^2$**

- **Je mehr Messwerte n ich habe, um so größer wird die mittlere quadratische Abweichung zwischen den Messwerten und der Funktion sein.**

- **Je mehr freie Parameter ich habe, um so kleiner wird mein $\chi^2$ werden.**

- **Freiheitsgrad = Anzahl der Messwerte – Anzahl der freien Parameter**
  **Ein $\chi^2$ von 1/Freiheitsgrad heißt, dass die Funktion die Messwerte im Rahmen der Fehler gut beschreibt.**
  **Beispiel: 4 Messwerte, Funktion=Gerade=>2 freie Parameter => 2 Freiheitsgrade**

# MAPLE Bibliotheken

**Prozeduren, die wir immer wieder verwenden wollen, können wir in eigenen MAPLE Bibliotheken ablegen:**

```
> restart:
> dv_maple:=table():
> # Weighted mean
> dv_maple[weigthed_mean]:=proc(data::list,sig::list)
    local nk, k, mean, e_mean, sig2:
    nk       := nops(data):
    sig2     := add(1/sig[k]^2,k=1..nk);
    mean     := add(data[k]/sig[k]^2,k=1..nk)/sig2;
    e_mean := 1/sqrt(sig2);
    return [mean,e_mean];
  end proc:
> save dv_maple, "/Users/stefan/lib/dv_maple.m";
```

$$a = \frac{\sum_{i=1}^{n} x_i \cdot 1/\sigma_i^2}{\sum_{i=1}^{n} 1/\sigma_i^2}$$

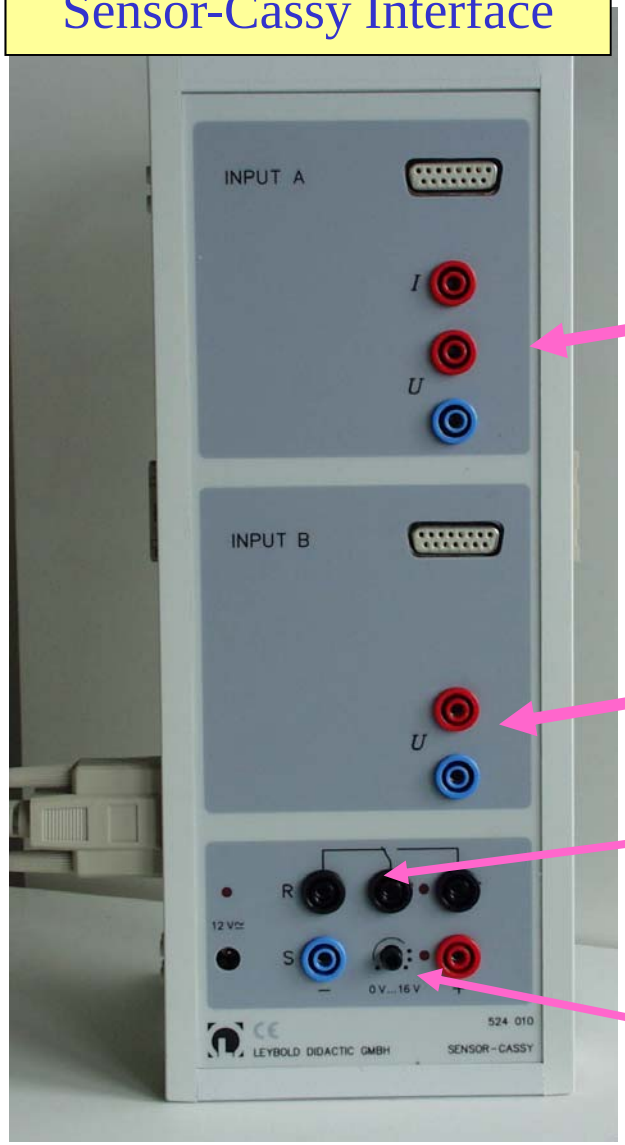
$$\sigma_a = \frac{1}{\sqrt{\sum_{i=1}^{n} 1/\sigma_i^2}}$$

**und anschließend, genau wie die MAPLE internen Pakete verwenden:**

```
> restart:
> libname:=libname,"/Users/stefan/lib/":
> with(dv_maple);
```

# Messgerät im Physikalischen Grundpraktikum

Sensor-Cassy Interface




Eingang A (I,U)

Eingang B (U)

Relais R

Spannungsquelle S (0 – 16V)

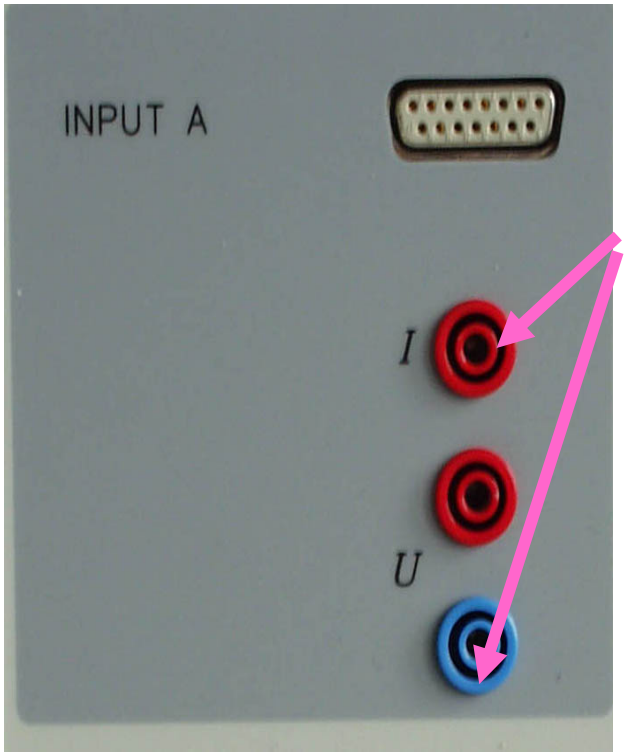
# Messgerät im Physikalischen Grundpraktikum


5 analoge Eingänge

2 analoge Spannungseingänge A und B:

- Auflösung: 12 Bit ( $2^{12} = 4096$)
- Messbereiche: ± 0,3/1/3/10/30/100 V
- Digitalisierung: ± 0,15 mV/…/ 48,8mV
- Eingangswiderstand: 1 MΩ
- Abtastrate: max. 200.000 Werte/s
  (=100.000 Werte/s pro Eingang)
- Anzahl Messwerte: max. 32000
  (= 16000/ Eingang)

# Messgerät im Physikalischen Grundpraktikum

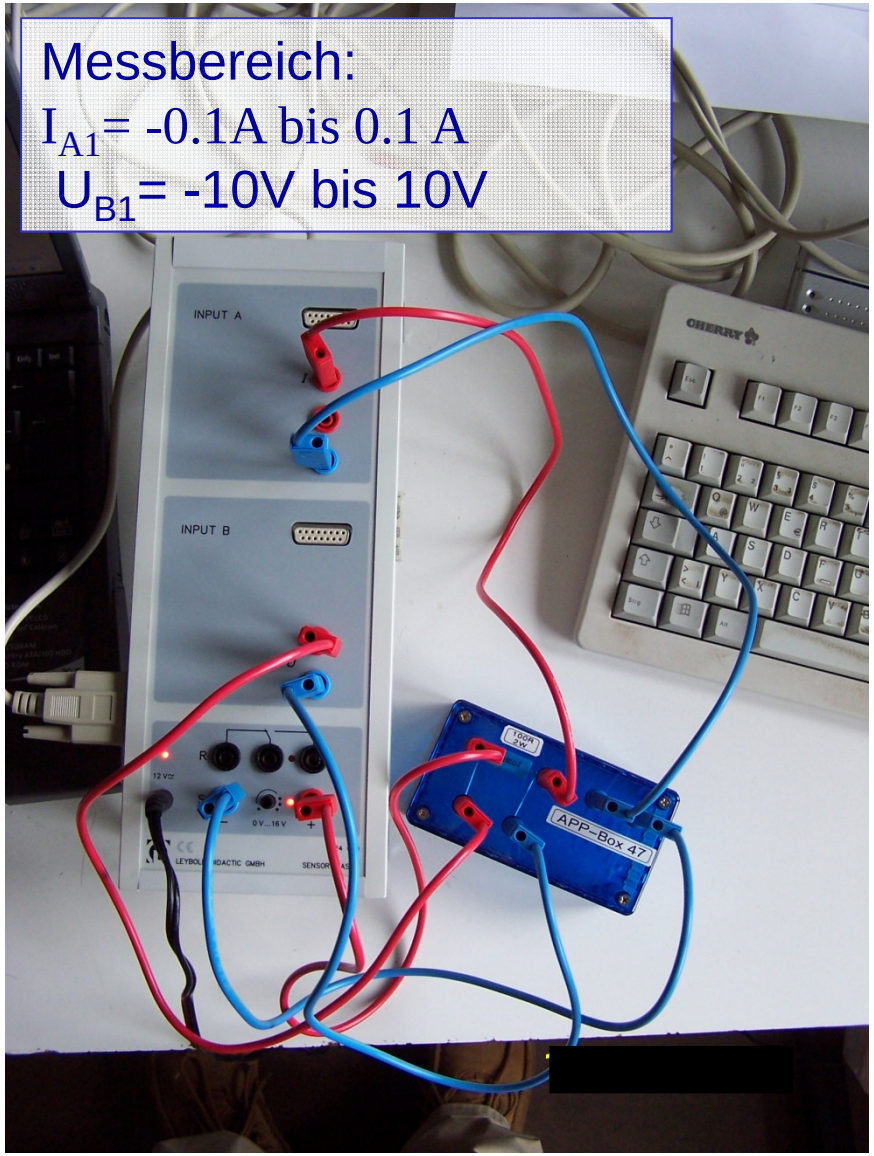


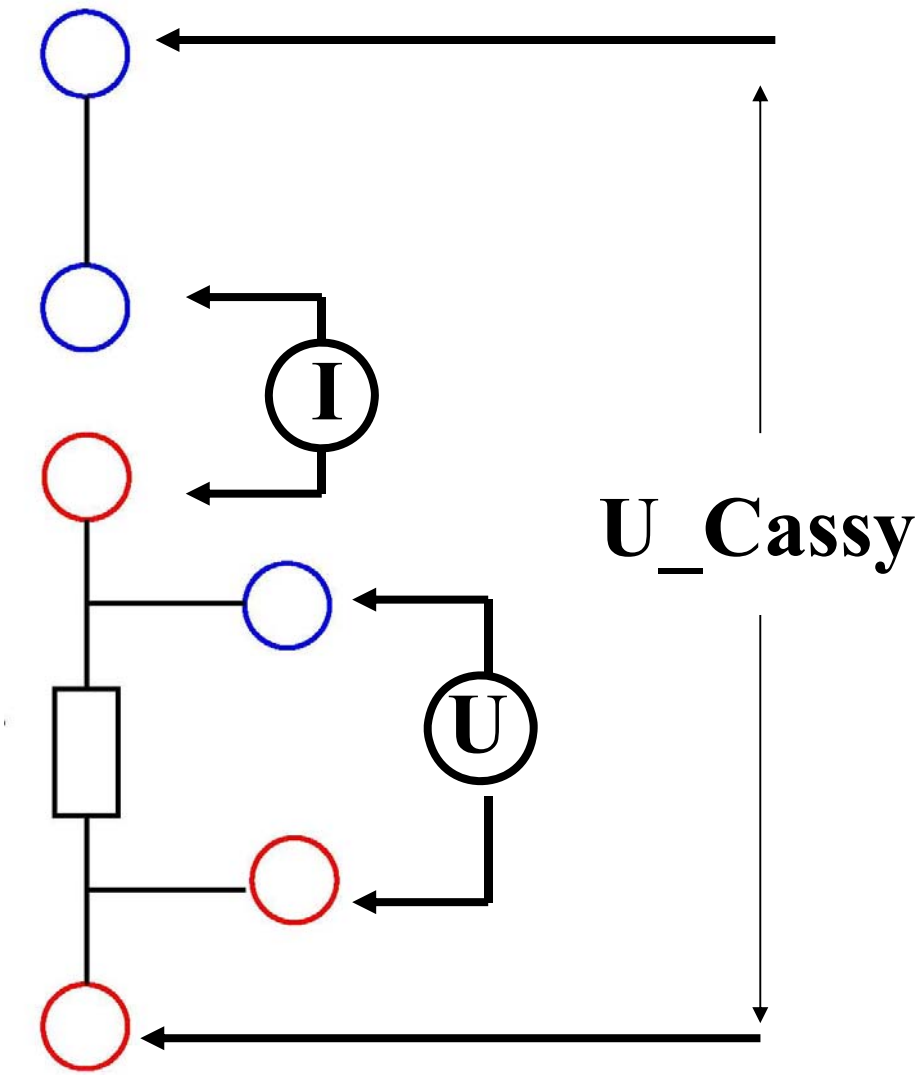

## Eingang A:

1 analoger Stromeingang :

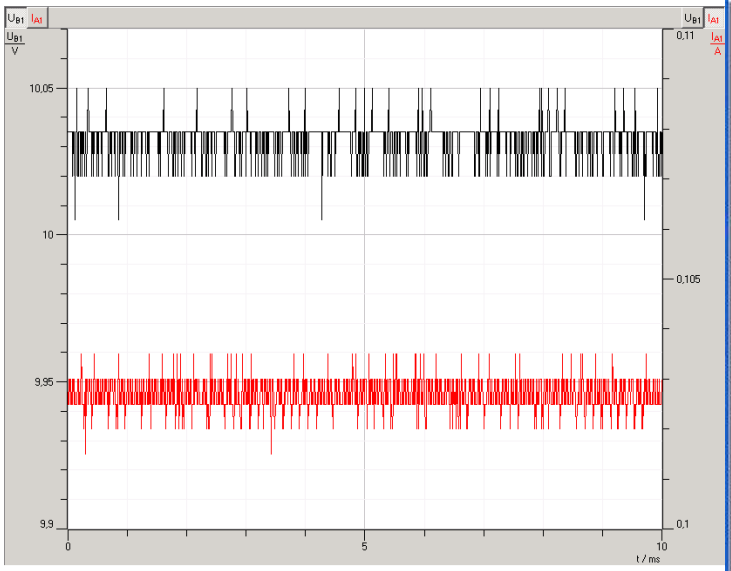
- Messbereiche: ± 0,1/0,3/1/3 A
- Digitalisierung: ± 0,05 mA/ … / 1,5 mA
- Eingangswiderstand: < 0,5 Ω

# Strom- Spannungsmessung im PGP



# Cassy-Lab Datei




100R_U_I_t_c.lab - XEmacs

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Open Dired Save Print Cut Copy Paste Undo Spell Replace Mail Info Compile Debug News

100R_U_I_t_c.lab

```
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1 0 0 0 1 1 3 0 0 0 0 0 0 0 0 2500 0 0 0 1 307 98 297 140 0.5 0 0 1
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A
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0 1 0 1
1 1 0 0 1 0 2 0 0 0 0 0 0 0 0 2500 0 0 0 1 608 102 297 140 0.5 0 0 1
Spannung
U_B1
V
-10 10 5 0 2 0 0 0 0 1 1 0 0 4.995 1 1
4 16001
1       0        0.04985 5
2       1E-5     0.0498  5.005
3       2E-5     0.04985 5
4       3E-5     0.04985 5
5       4E-5     0.04985 5
6       5E-5     0.04985 5
7       6E-5     0.04985 5
8       7E-5     0.04985 5
9       8E-5     0.04985 5
10      9E-5     0.04985 5
```
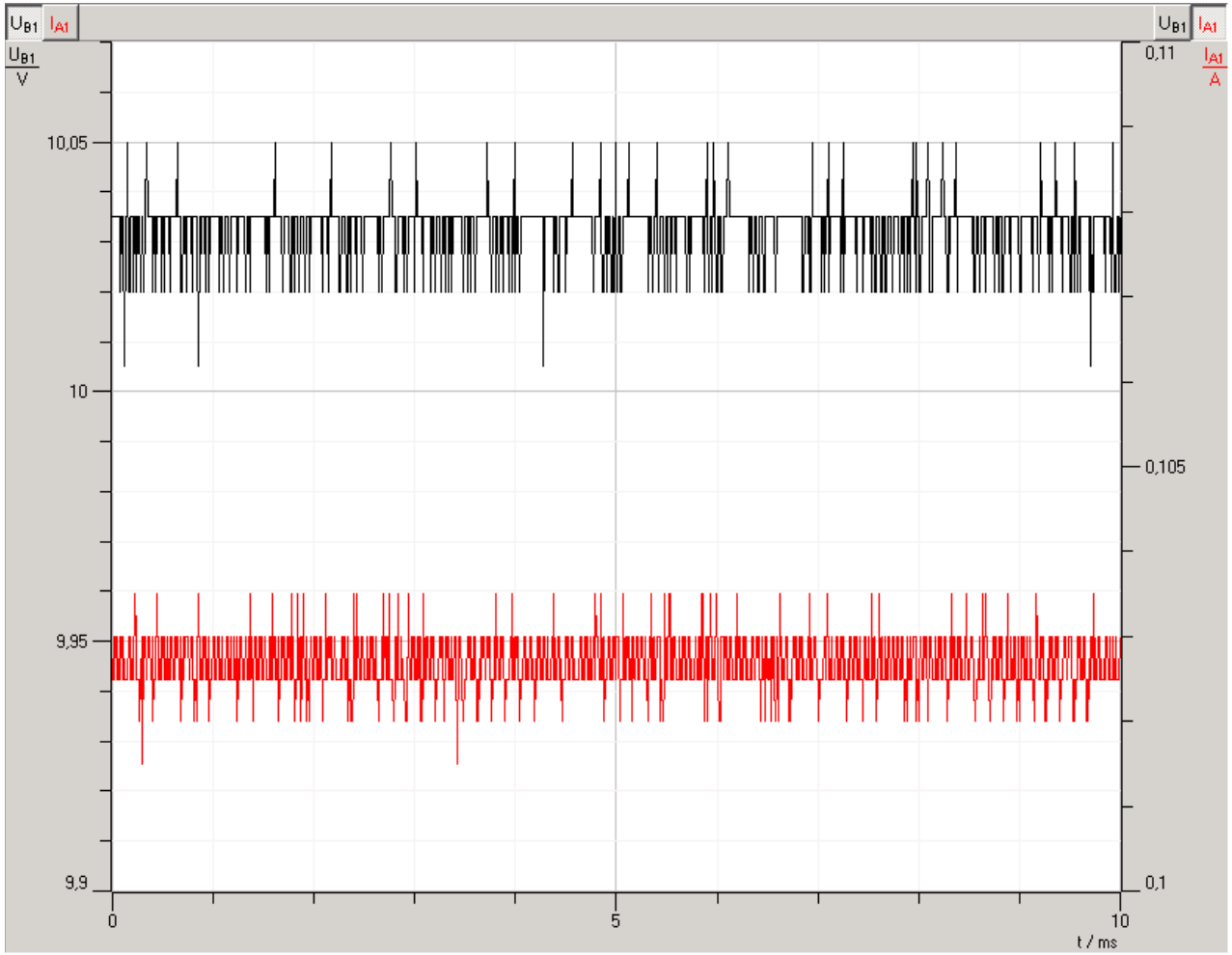

# Cassy-Lab Datei ↔ Maple-Auswertung
# Bestimmung des Ohmschen Widerstandes

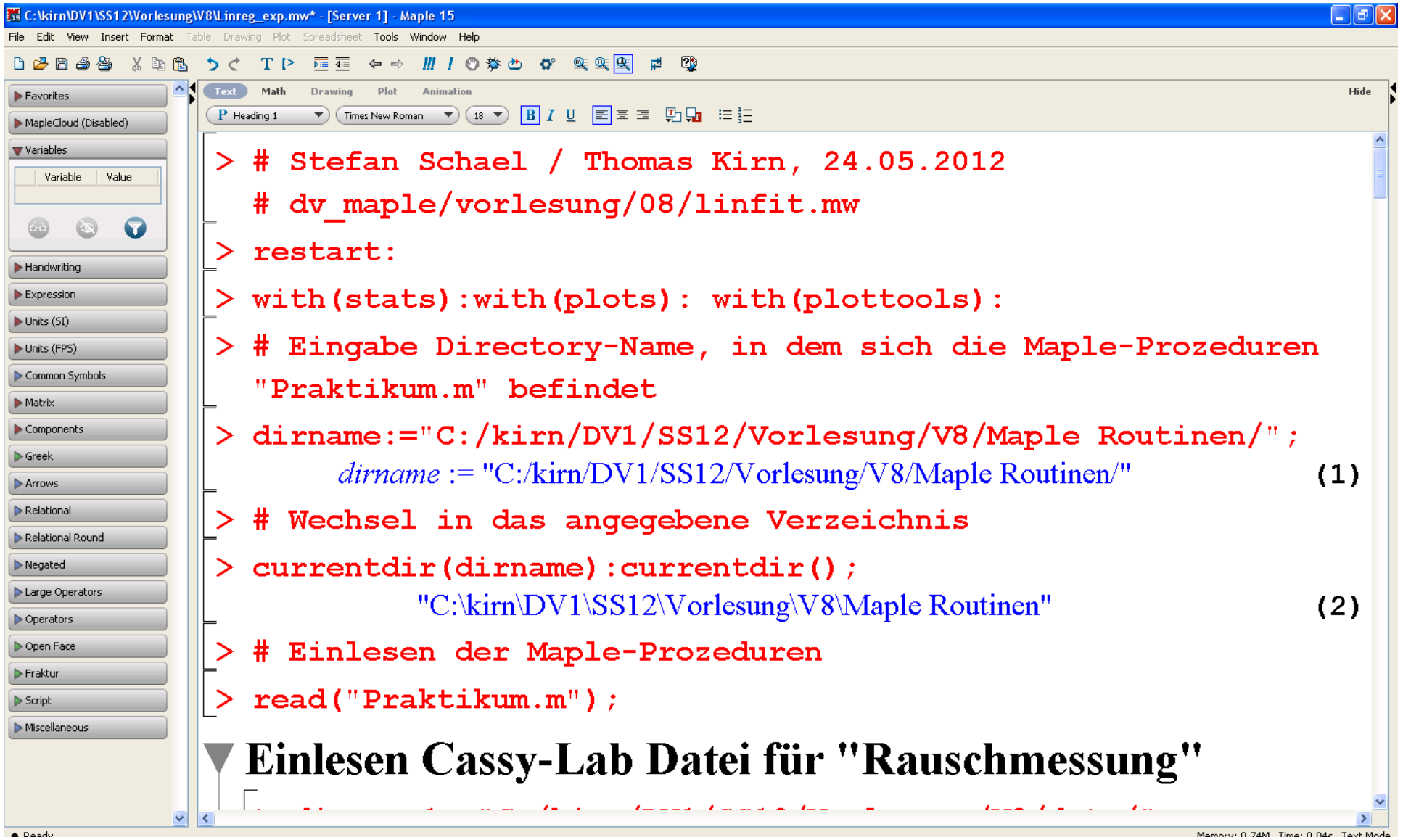



$$< x >= \frac{1}{n} \cdot \sum_{i=1}^{n} x_i$$

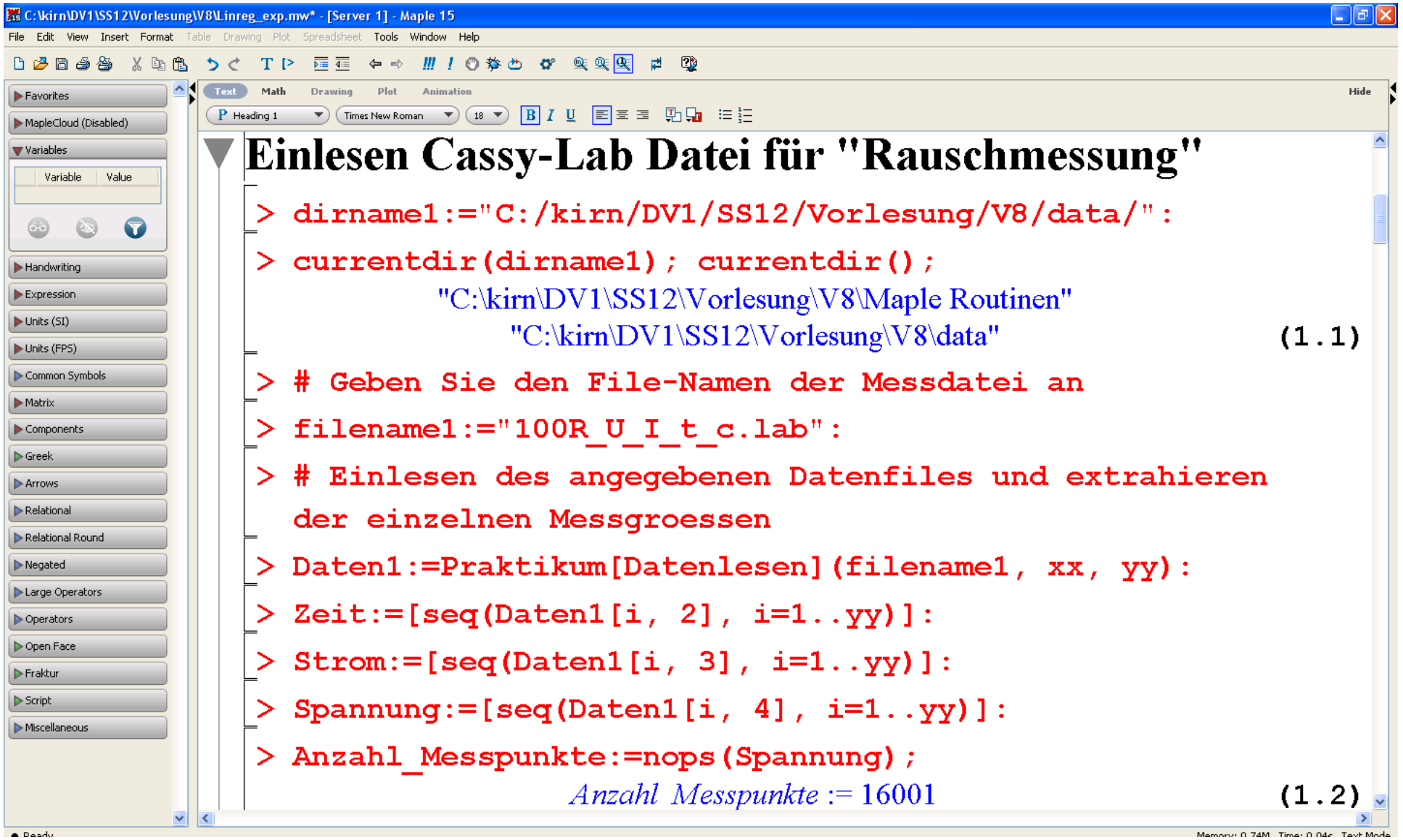
$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - < x >)^2}{n-1}}$$

$$\sigma_{<x>} = \frac{\sigma}{\sqrt{n}}$$

# Cassy-Lab Datei ↔ Maple-Auswertung



# Einlesen Cassy-Lab Dateien in Maple mit Praktikumsbibliothek



# Verteilung der Strommesswerte

C:\kirn\DV1\SS12\Vorlesung\V8\Linreg_exp_test.mw* - [Server 1] - Maple 15

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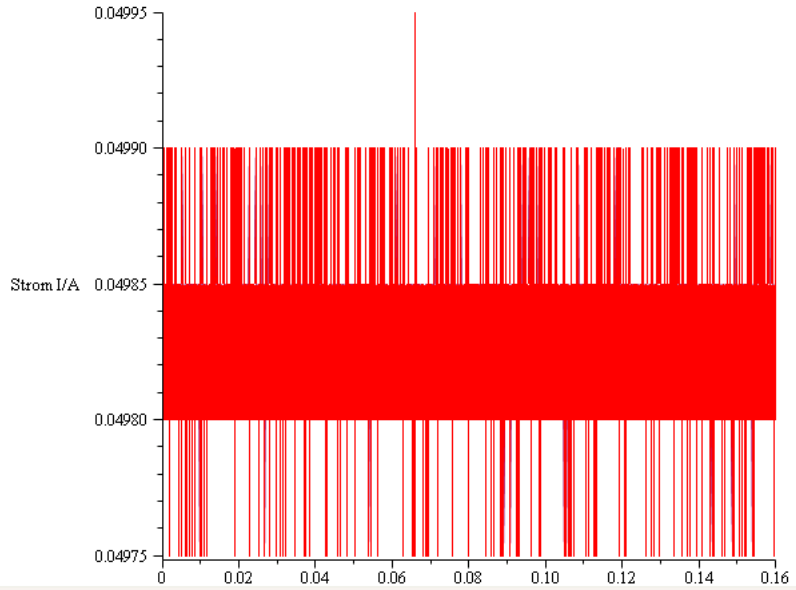
Text Math Drawing Plot Animation Hide

Maple Plot | Times New Roman | 12

## Verteilung Strommessung

```
> plot_it1:=Praktikum[Plotdata]([Zeit, Strom,color=blue]):
> plot_it2:=plot(Zeit, Strom, color=red):
> display({plot_it1},{plot_it2}, labels=["Zeit t/s",
  "Strom I/A"]);
```




Favorites
MapleCloud (Disabled)
Variables

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|---|---|
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| Daten1 | table([(33... |
| dirname | "C:/kirn/D... |
| dirname1 | "C:/kirn/D... |
| filename1 | "100R_U_... |
| plot_it | PLOT(CU... |
| plot_it1 | PLOT(CU... |
| plot_it2 | PLOT(CU... |
| Spannung | [5, 5.005... |
| Strom | [.4985e-1... |
| Zeit | [0, .1e-4,... |

Handwriting
Expression
Units (SI)
Units (FPS)
Common Symbols
Matrix
Components
Greek
Arrows
Relational
Relational Round
Negated
Large Operators
Operators
Open Face

Ready

Memory: 99.98M Time: 7.12s Text Mode

# Verteilung der Strommesswerte

C:\kirn\DV1\SS12\Vorlesung\V8\Linreg_exp_test.mw* - [Server 1] - Maple 15

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Text Math Drawing Plot Animation Hide

Maple Plot | Times New Roman | 12

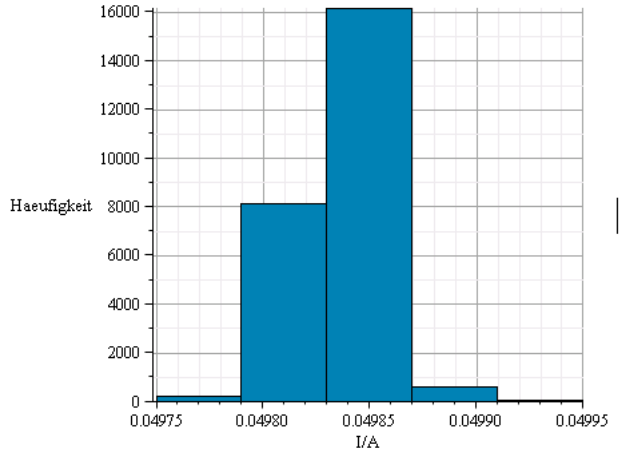
```
> MeanStrom:=Praktikum[Mean](Strom);
Mean =    4.9834E-02 +/-    2.0754E-07    (Mittlerer quadratischer Fehler des Mittelwertes)
Rms  =    2.6252E-05                      (Standardabweichung)
```

$$MeanStrom := [0.04983428223,\ 2.075366333\ 10^{-7},\ 0.00002625235871] \qquad (2.1)$$

```
> with(Statistics):
> Mean(Strom);sqrt(Variance(Strom));sqrt(Variance(Strom)/nops(Strom));
```

$$0.04983428223$$
$$0.0000262523587495735$$
$$2.07536633582789\ 10^{-7} \qquad (2.2)$$

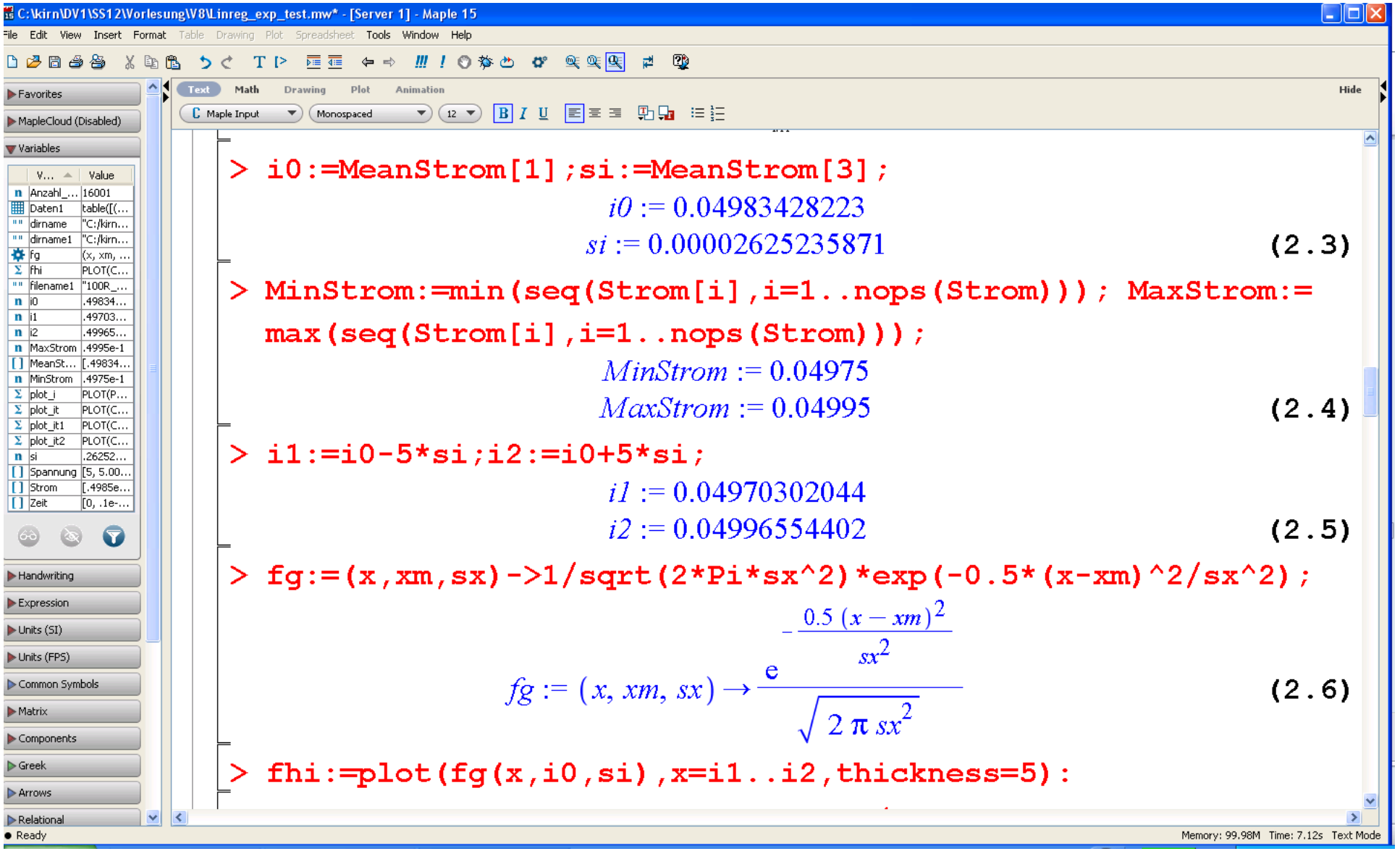
```
> plot_i:=Histogram(Strom, bincount=5):
> display({plot_i},labels=["I/A", "Haeufigkeit"],gridlines=true);
```




Ready

Memory: 99.98M Time: 7.12s Text Mode

# Verteilung der Strommesswerte



# Verteilung der Strommesswerte

C:\kirn\DV1\SS12\Vorlesung\V8\Linreg_exp_test.mw - [Server 1] - Maple 15

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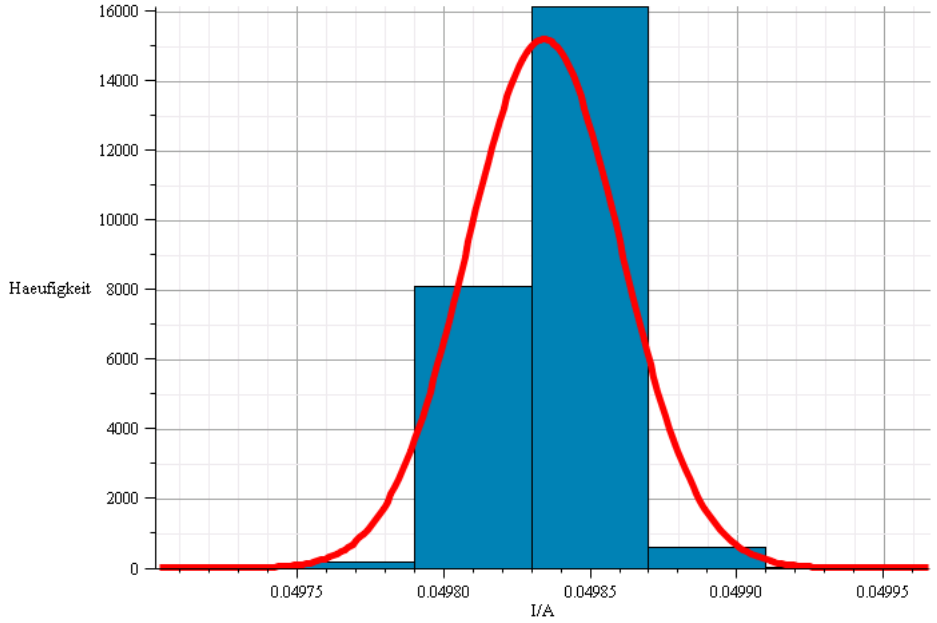
Maple Plot | Times New Roman | 12

```
> fhi:=plot(fg(x,i0,si),x=i1..i2,thickness=5):
> display({plot_i},{fhi},labels=["I/A", "Haeufigkeit"],
  gridlines=true);
```



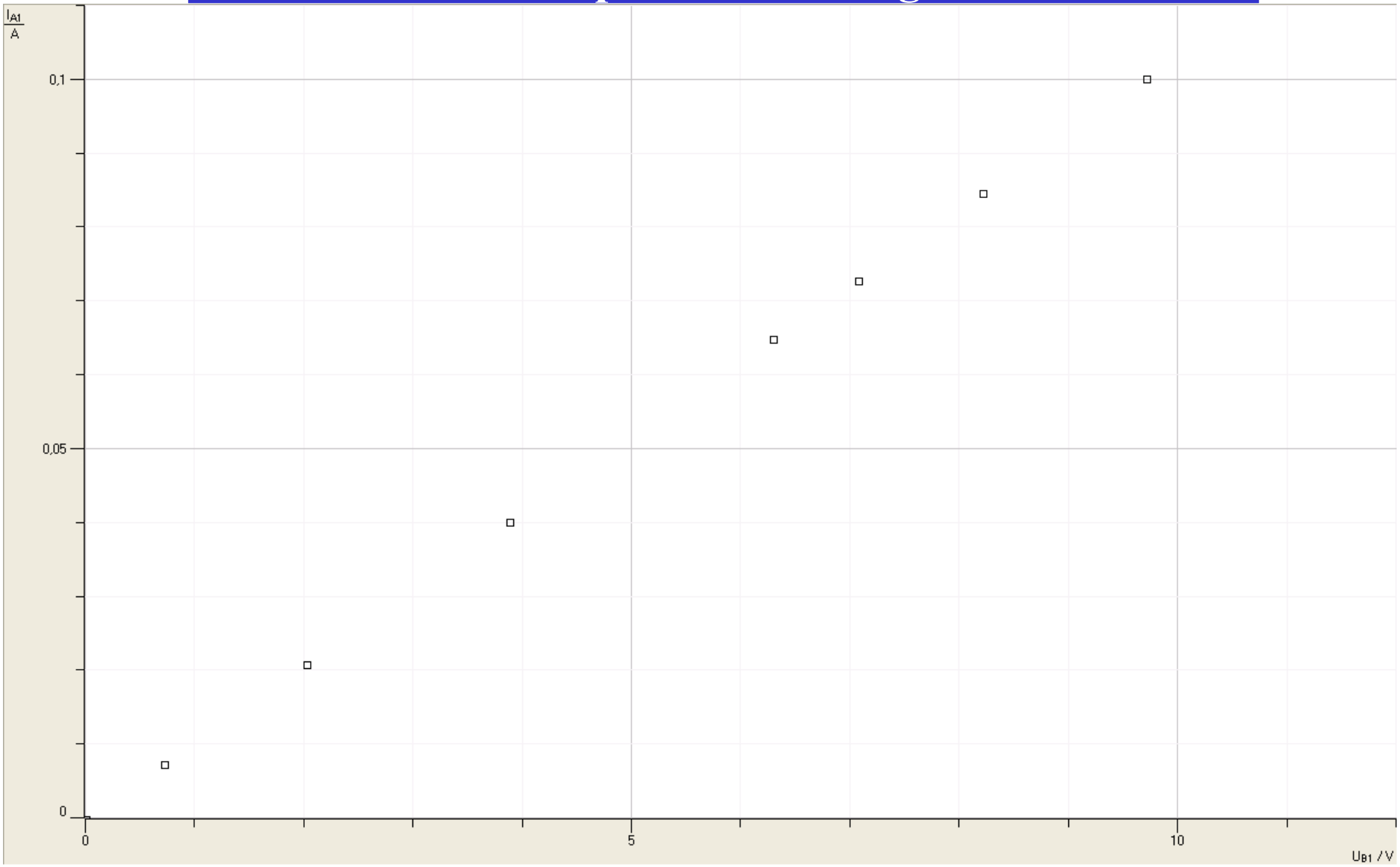

```
> rel_err_i:=si/i0;
```

$$rel\_err\_i := 0.0005267931539 \qquad (2.7)$$

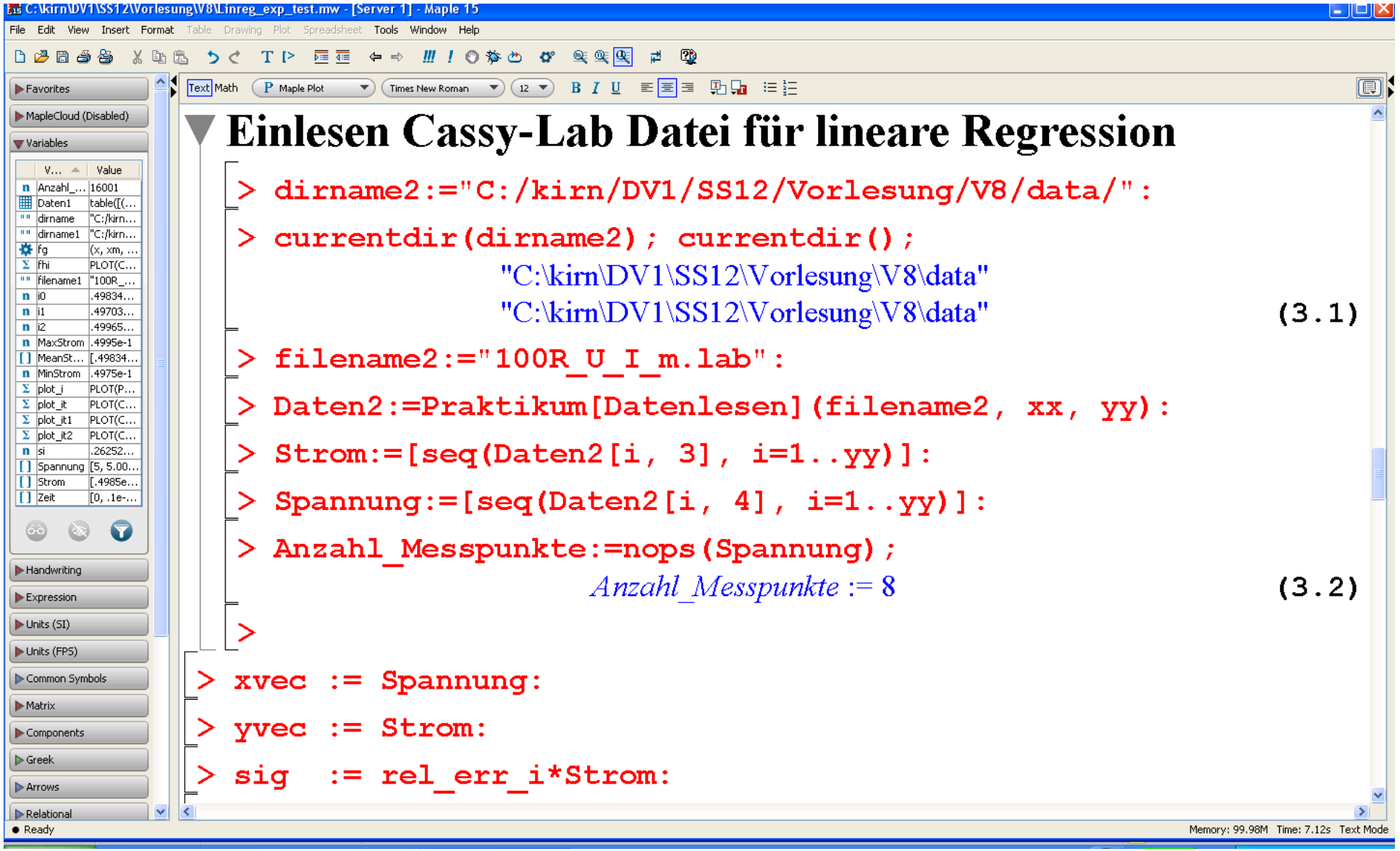
Ready

Memory: 99.98M Time: 7.12s Text Mode

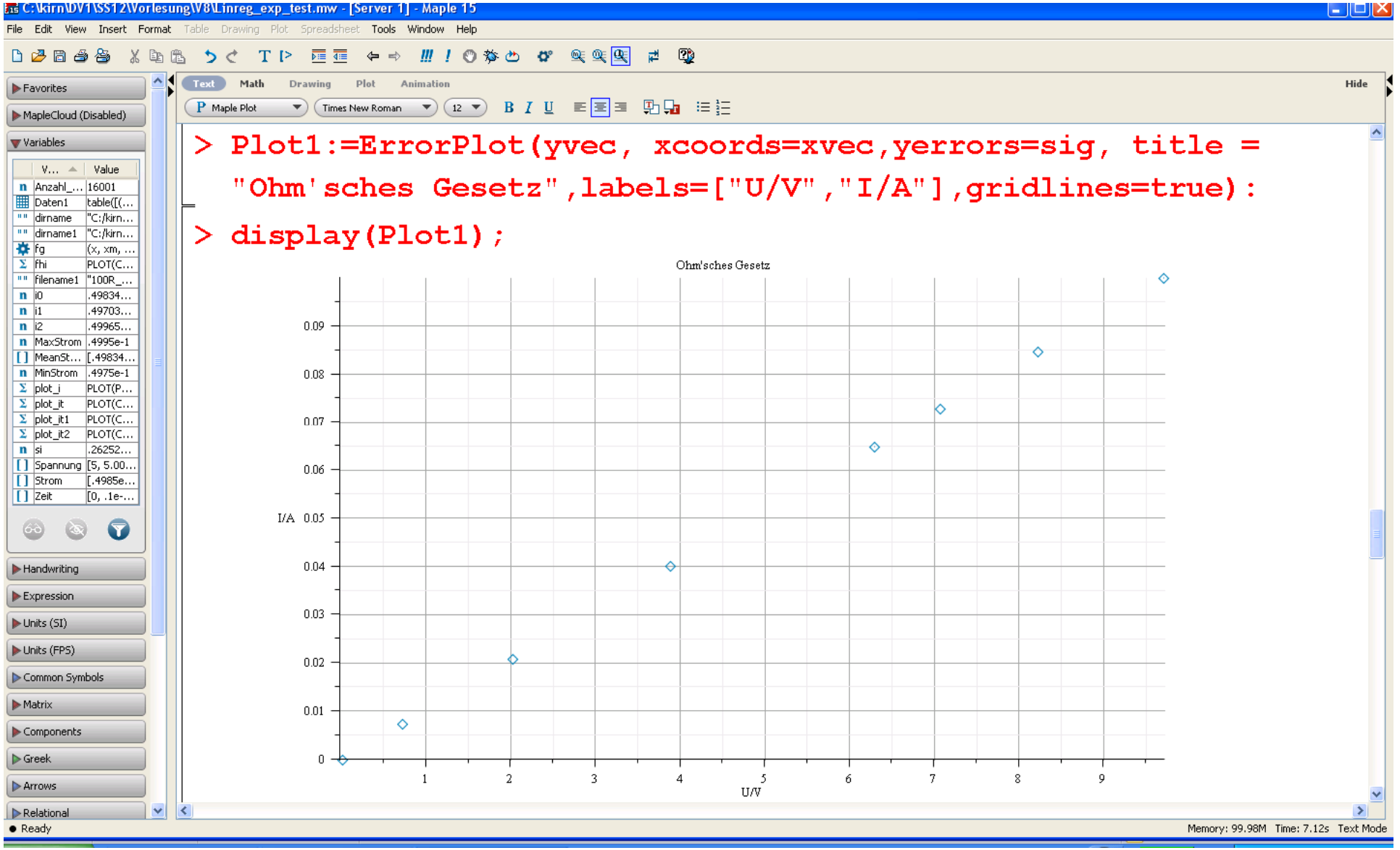
# Lineare Regression mit Cassy-Lab-Dateien via Maple-Auswertung



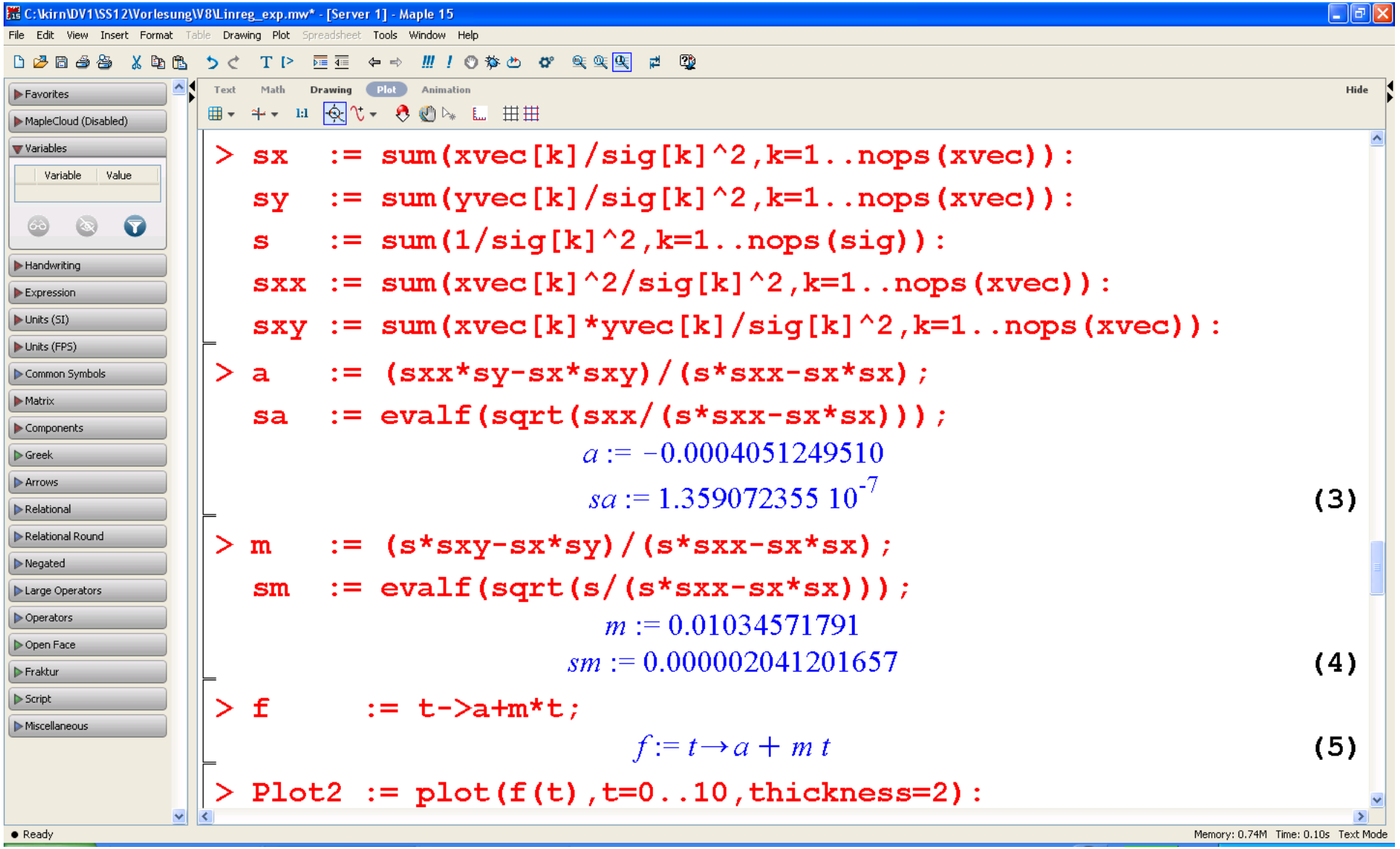
# Lineare Regression mit Cassy-Lab-Dateien mit Maple



# Lineare Regression mit Cassy-Lab-Dateien mit Maple



# Lineare Regression mit Cassy-Lab-Dateien mit Maple



# Lineare Regression mit Cassy-Lab-Dateien mit Maple

C:\kirn\DV1\SS12\Vorlesung\V8\Linreg_exp_test.mw - [Server 1] - Maple 15

File Edit View Insert Format Table Drawing Plot Spreadsheet Tools Window Help

Favorites
MapleCloud (Disabled)
Variables

| V... | Value |
|---|---|
| Anzahl_... | 16001 |
| Daten1 | table([(... |
| dirname | "C:/kirn... |
| dirname1 | "C:/kirn... |
| fg | (x, xm, ... |
| fhi | PLOT(C... |
| filename1 | "100R_... |
| i0 | .49834... |
| i1 | .49703... |
| i2 | .49965... |
| MaxStrom | .4995e-1 |
| MeanSt... | [.49834... |
| MinStrom | .4975e-1 |
| plot_i | PLOT(P... |
| plot_it | PLOT(C... |
| plot_it1 | PLOT(C... |
| plot_it2 | PLOT(C... |
| si | .26252... |
| Spannung | [5, 5.00... |
| Strom | [.4985e... |
| Zeit | [0, .1e-... |

Handwriting
Expression
Units (SI)
Units (FPS)
Common Symbols
Matrix
Components
Greek
Arrows
Relational

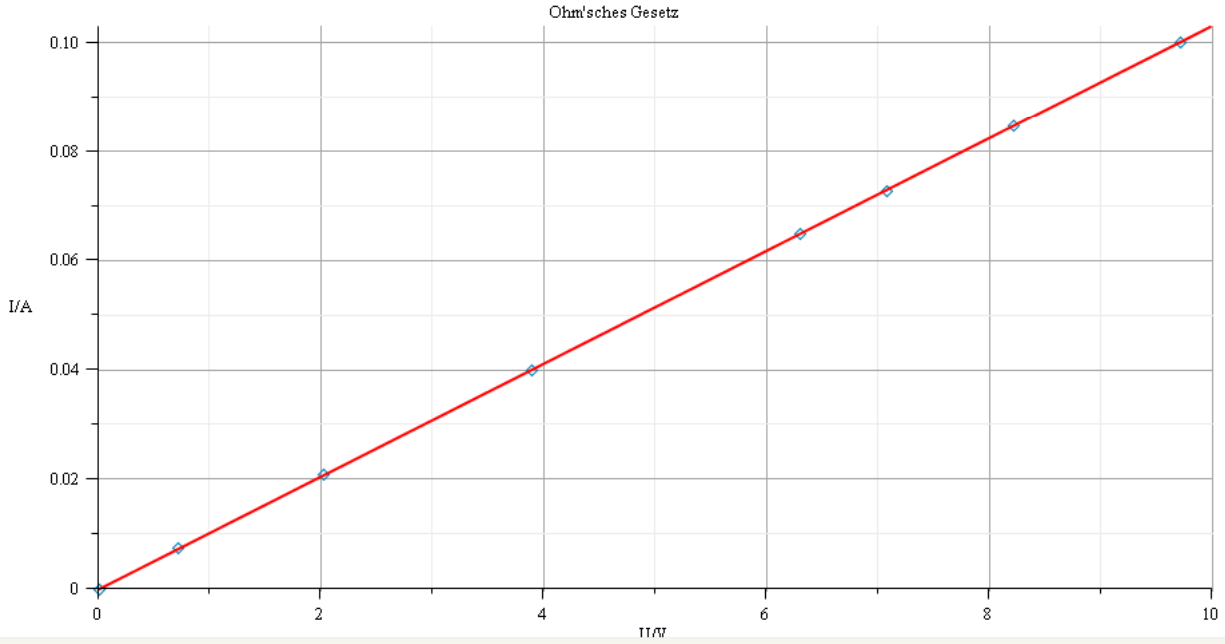
Text Math Drawing Plot Animation Hide

Maple Plot Times New Roman 12

```
> f      := t->a+m*t;
```

$$f := t \to a + m\,t \qquad (5)$$

```
> Plot2 := plot(f(t),t=0..10,thickness=2):
> display(Plot1,Plot2,title = "Ohm'sches Gesetz",labels=
  ["U/V","I/A"],gridlines=true);
```

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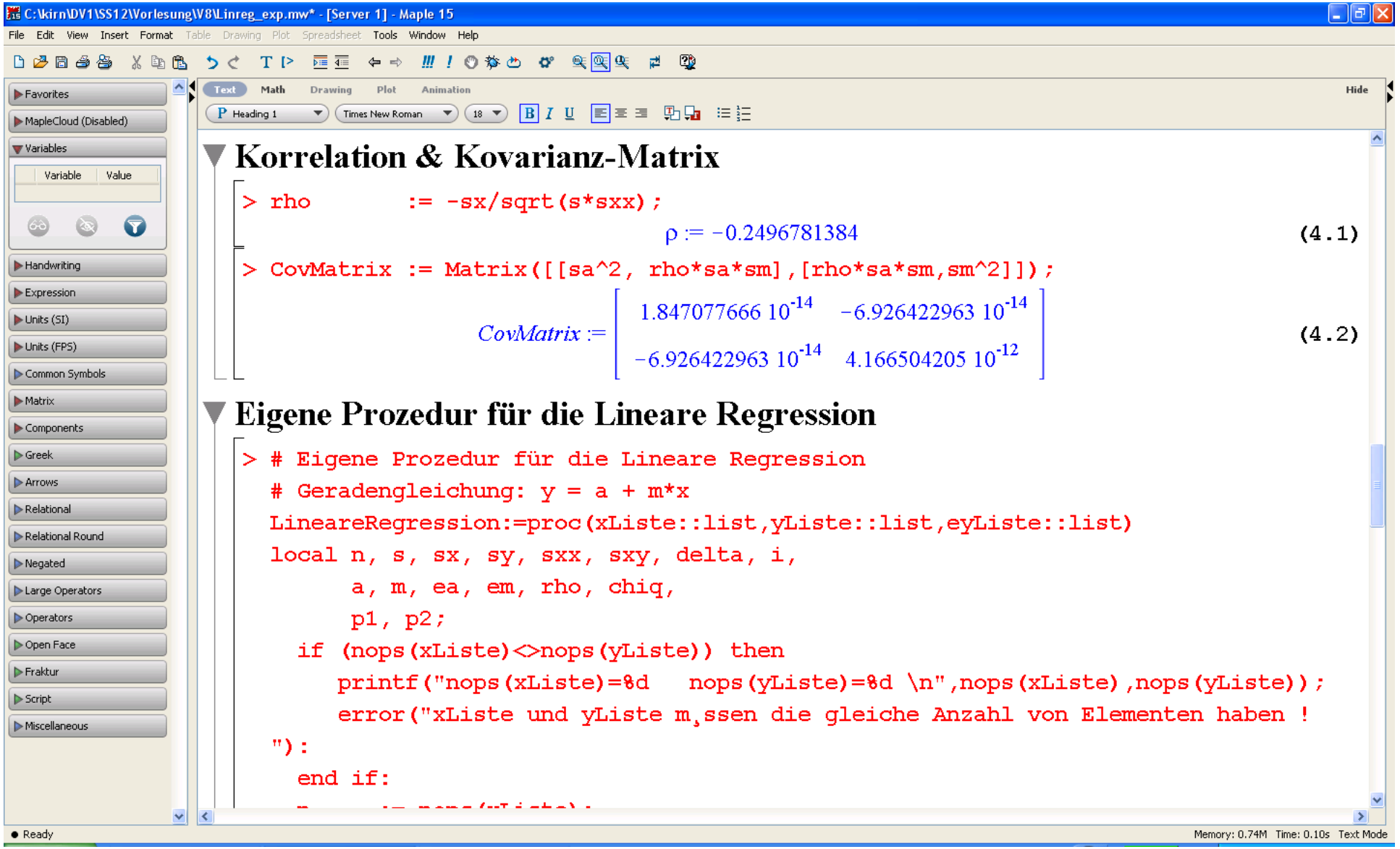
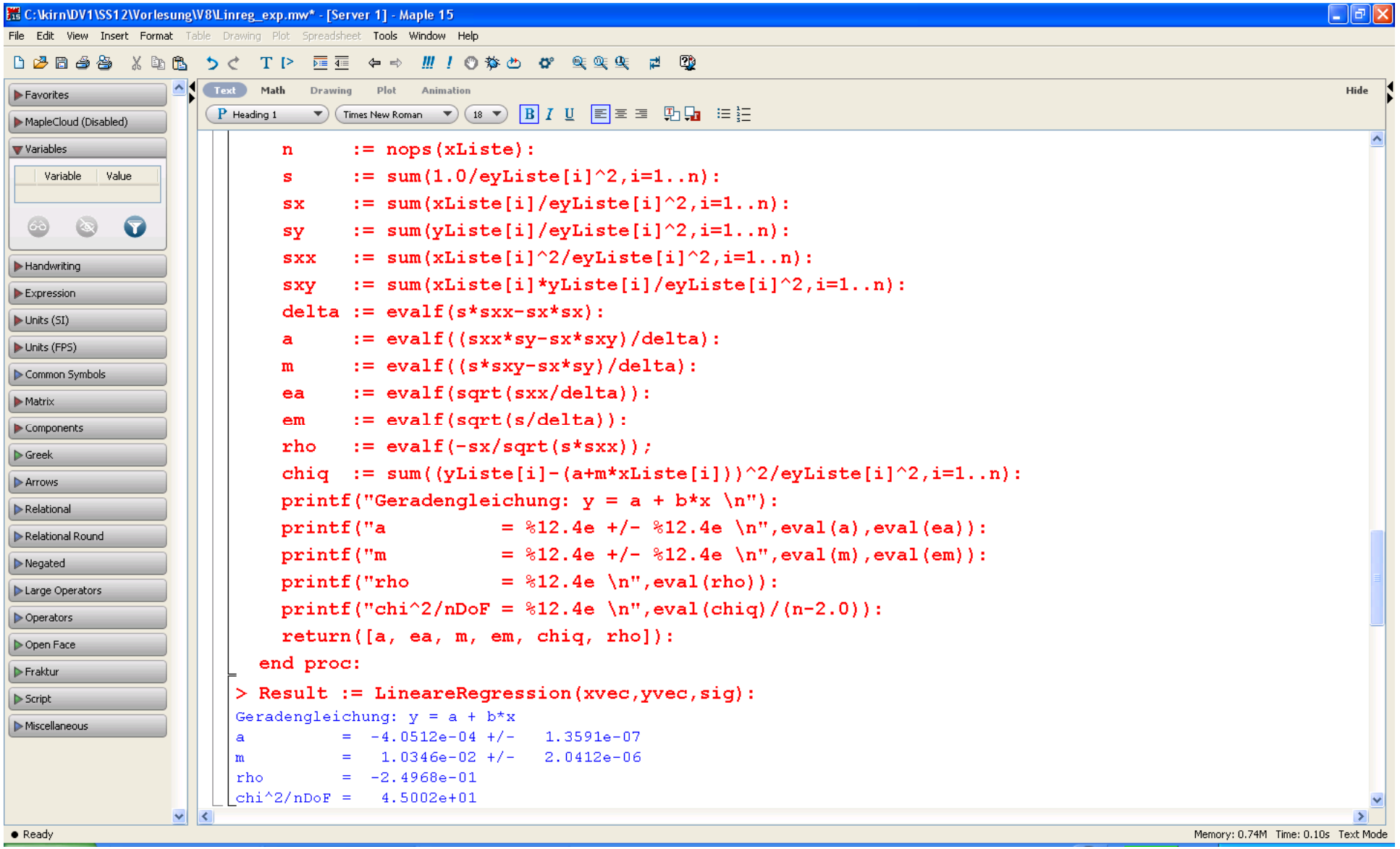
Memory: 99.98M Time: 7.12s Text Mode

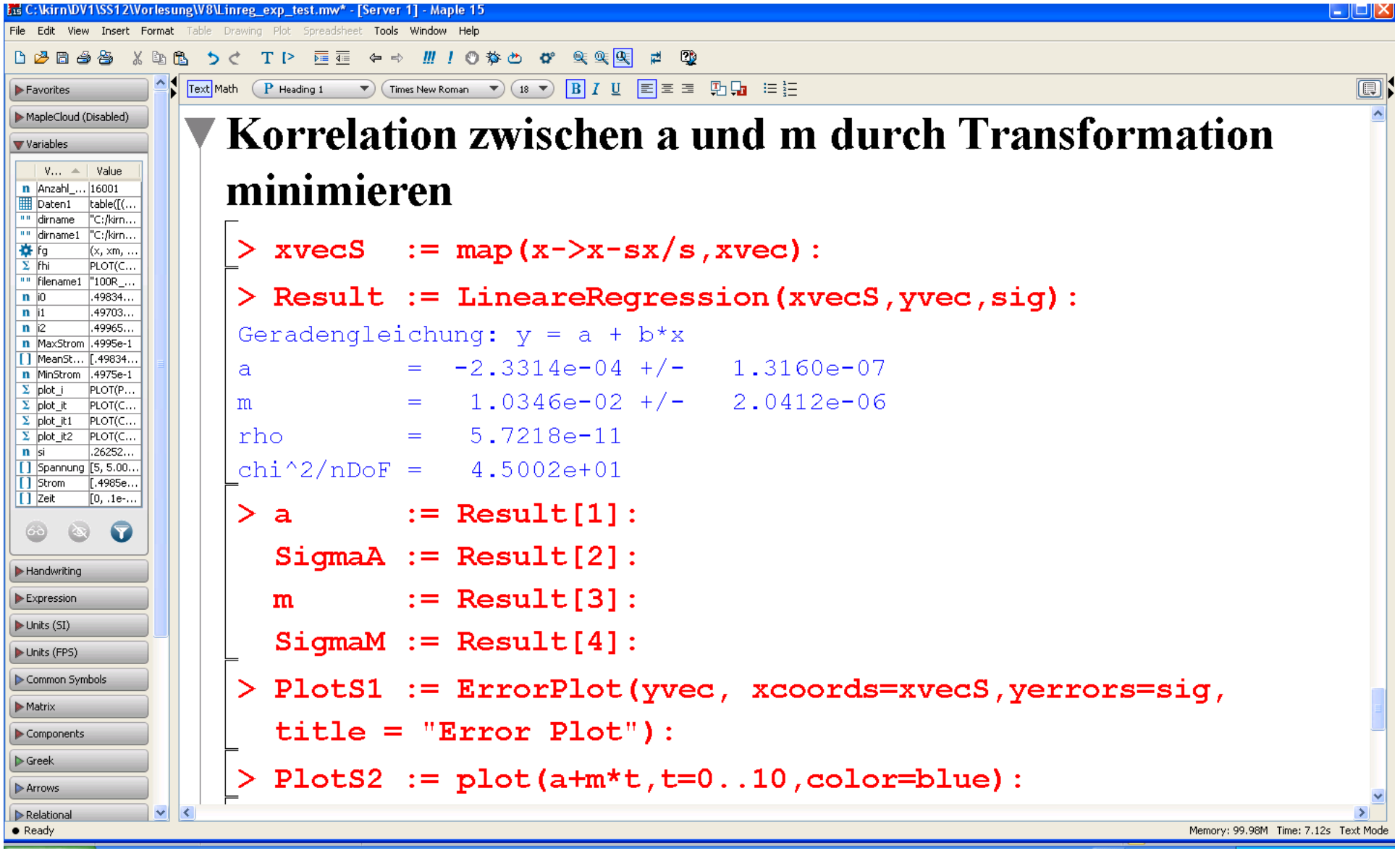
# Lineare Regression mit Cassy-Lab-Dateien mit Maple



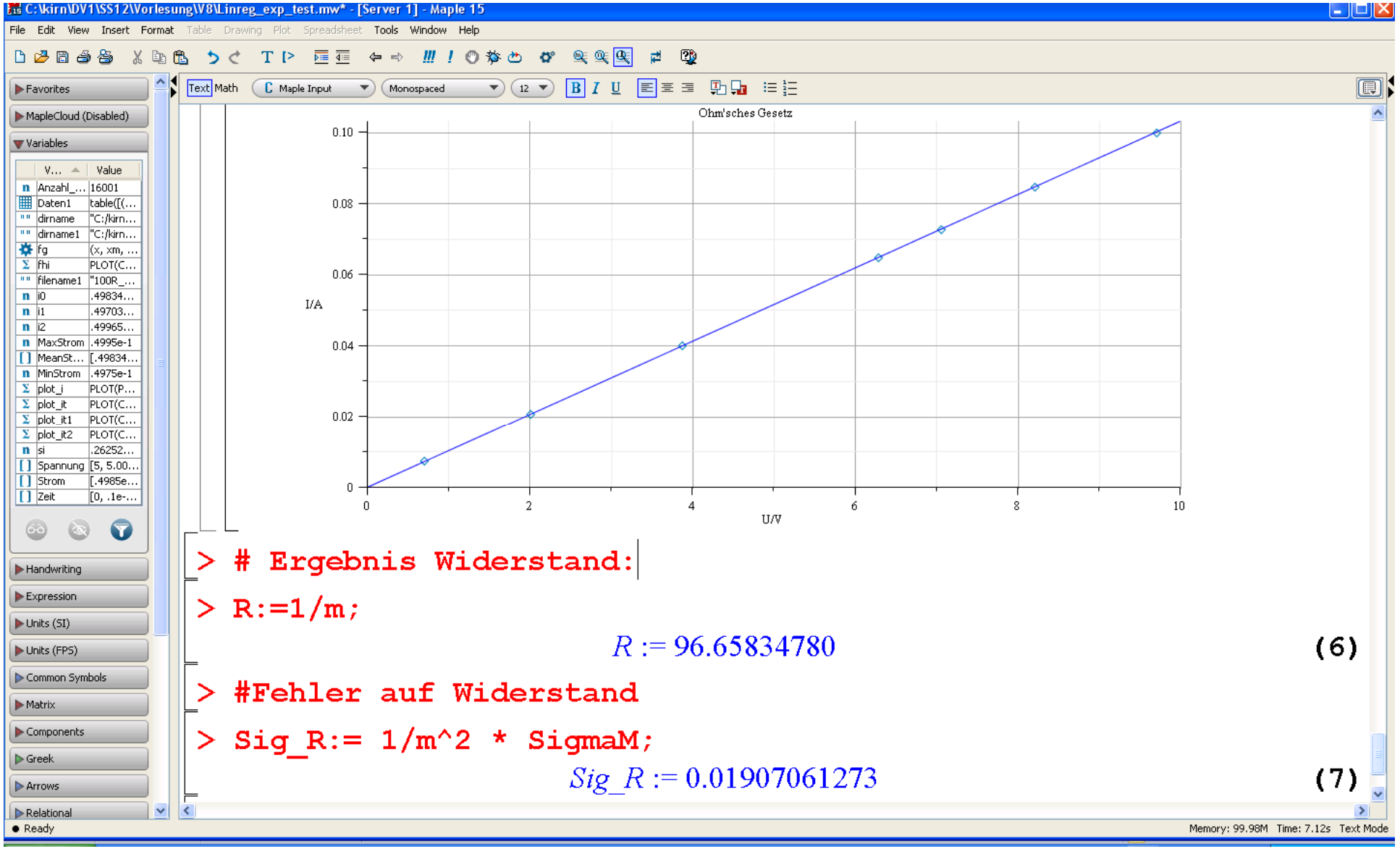
# Lineare Regression mit Cassy-Lab-Dateien mit Maple



# Lineare Regression mit Cassy-Lab-Dateien mit Maple



# Lineare Regression mit Cassy-Lab-Dateien mit Maple



# Anhang 1

$$\sigma_a^2 = \sum_{i=1}^{n} \sigma_i^2 \left( \frac{\partial a}{\partial y_i} \right)^2 \qquad \frac{\partial a}{\partial y_i} = \frac{1}{\Delta \sigma_i^2} \left( S_{xx} - S_x \cdot x_i \right)$$

$$\Rightarrow \sigma_a^2 = \sum_{i=1}^{n} \sigma_i^2 \left( \frac{1}{\Delta \sigma_i^2} \left( S_{xx} - S_x \cdot x_i \right) \right)^2 = \sum_{i=1}^{n} \frac{1}{\Delta^2 \sigma_i^2} \left( S_{xx}^2 - 2 S_{xx} S_x x_i + S_x^2 x_i^2 \right)$$

$$= \frac{1}{\Delta^2} \left( S_{xx}^2 S - 2 S_{xx} S_x^2 + S_x^2 S_{xx} \right) = \frac{1}{\Delta^2} \left( S_{xx}^2 S - S_{xx} S_x^2 \right)$$

$$= \frac{S_{xx}}{\Delta^2} \left( S \cdot S_{xx} - S_x S_x \right) = \frac{S_{xx}}{\Delta}$$

$$S_y \equiv \sum_{i=1}^{n} \frac{y_i}{\sigma_i^2} \qquad S_x \equiv \sum_{i=1}^{n} \frac{x_i}{\sigma_i^2} \qquad S_{xy} \equiv \sum_{i=1}^{n} \frac{x_i y_i}{\sigma_i^2}$$

$$S_{xx} = \sum_{i=1}^{n} \frac{x_i x_i}{\sigma_i^2} \qquad S \equiv \sum_{i=1}^{n} \frac{1}{\sigma_i^2} \qquad \Delta = S \cdot S_{xx} - S_x S_x$$